# Computer Vision 2 – Lecture 6

## Beyond Kalman Filters (09.05.2016)

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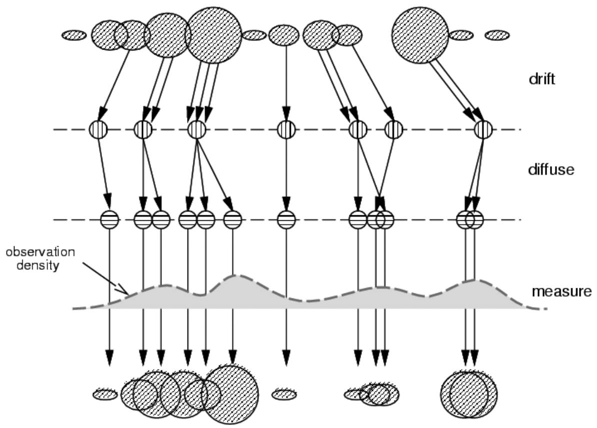
# Content of the Lecture

- Single-Object Tracking
- Bayesian Filtering
  - Kalman Filters, EKF
  - Particle Filters
- Multi-Object Tracking
- Visual Odometry
- Visual SLAM & 3D Reconstruction



# Today: Beyond Gaussian Error Models

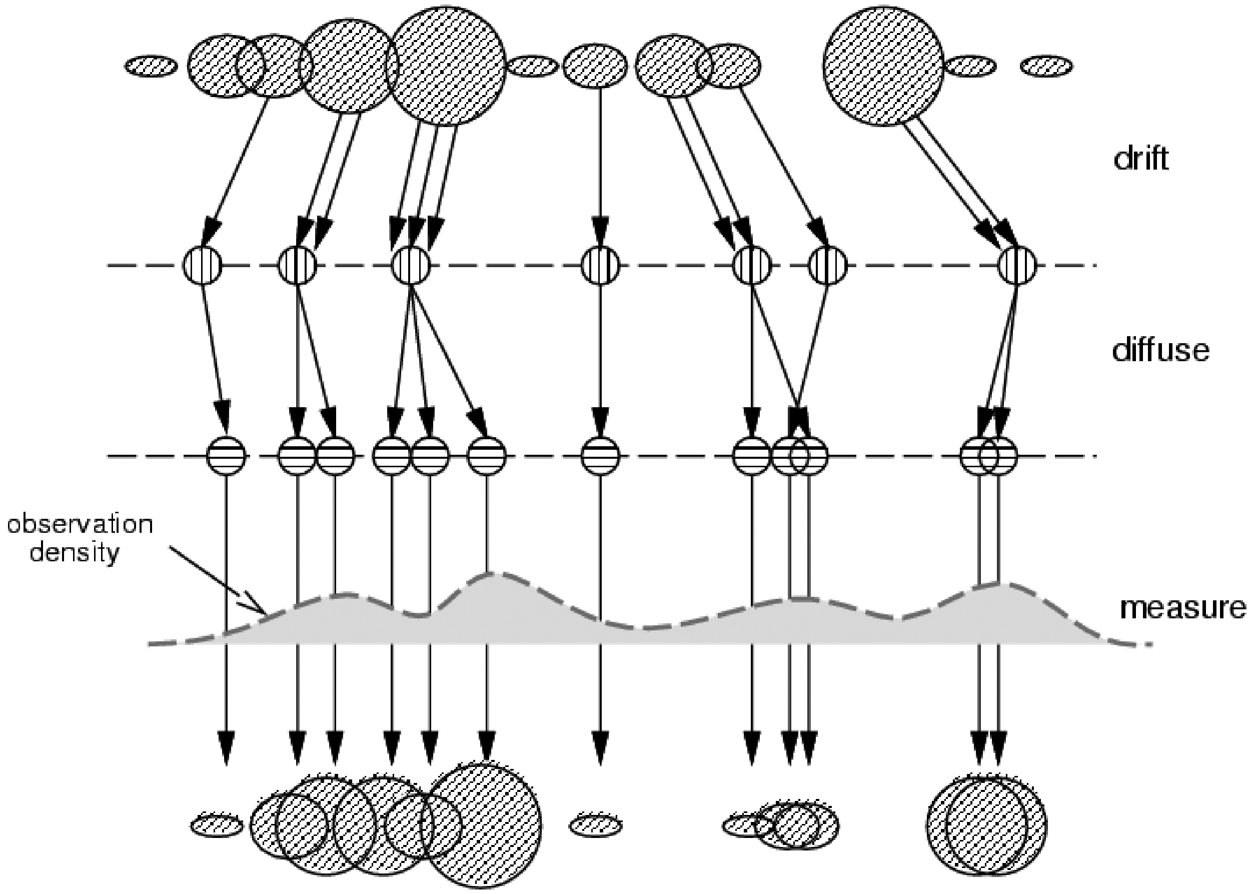


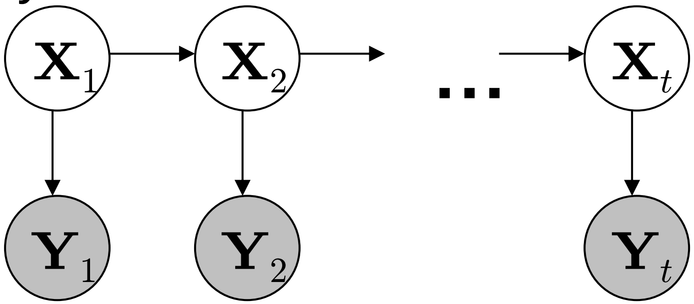

Figure from Isard & Blake

# Topics of This Lecture

- Recap: Kalman Filter
  - Basic ideas
  - Limitations
  - Extensions
- Particle Filters
  - Basic ideas
  - Propagation of general densities
  - Factored sampling
- Case study
  - Detector Confidence Particle Filter
  - Role of the different elements

# Recap: Tracking as Inference

- Inference problem
  - The hidden state consists of the true parameters we care about, denoted $\mathbf{X}$.
  - The measurement is our noisy observation that results from the underlying state, denoted $\mathbf{Y}$.
  - At each time step, state changes (from $\mathbf{X}_{t\text{-}1}$ to $\mathbf{X}_t$) and we get a new observation $\mathbf{Y}_t$.
- Our goal: recover most likely state $\mathbf{X}_t$ given
  - All observations seen so far.
  - Knowledge about dynamics of state transitions.

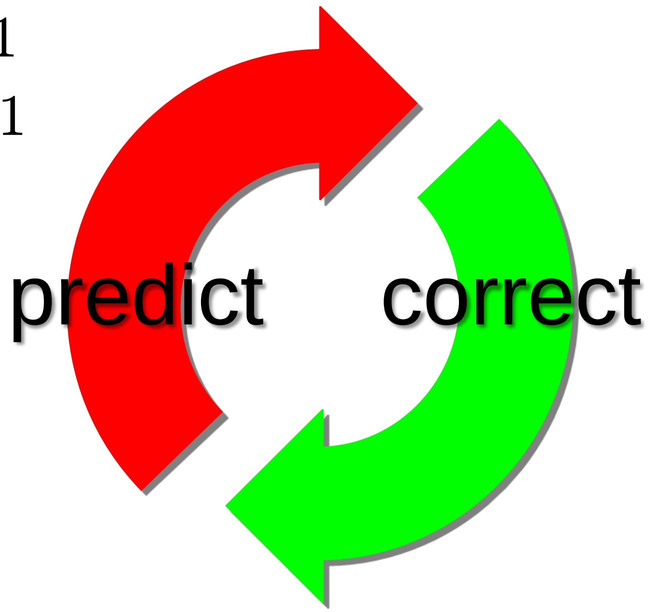





Slide credit: Kristen Grauman

# Recap: Tracking as Induction

- Base case:
  - Assume we have initial prior that predicts state in absence of any evidence: $P(\mathbf{X}_0)$
  - At the first frame, *correct* this given the value of $\mathbf{Y}_0 = \mathbf{y}_o$
- Given corrected estimate for frame $t$:
  - Predict for frame $t+1$
  - Correct for frame $t+1$





Slide credit: Svetlana Lazebnik

# Recap: Prediction and Correction

- Prediction:

$$P(X_t \mid y_0,\ldots,y_{t-1}) = \int \underbrace{P(X_t \mid X_{t-1})}_{\text{Dynamics model}} \underbrace{P(X_{t-1} \mid y_0,\ldots,y_{t-1})}_{\text{Corrected estimate from previous step}} dX_{t-1}$$

- Correction:

$$P(X_t \mid y_0,\ldots,y_t) = \frac{\overbrace{P(y_t \mid X_t)}^{\text{Observation model}} \overbrace{P(X_t \mid y_0,\ldots,y_{t-1})}^{\text{Predicted estimate}}}{\int P(y_t \mid X_t) P(X_t \mid y_0,\ldots,y_{t-1}) dX_t}$$



Slide credit: Svetlana Lazebnik

# Recap: Linear Dynamic Models

- Dynamics model
  - State undergoes linear transformation $D_t$ plus Gaussian noise

$$\boldsymbol{x}_t \sim N\left(\boldsymbol{D}_t \boldsymbol{x}_{t-1}, \Sigma_{d_t}\right)$$

- Observation model
  - Measurement is linearly transformed state plus Gaussian noise

$$\boldsymbol{y}_t \sim N\left(\boldsymbol{M}_t \boldsymbol{x}_t, \Sigma_{m_t}\right)$$



Slide credit: Svetlana Lazebnik, Kristen Grauman

# Recap: Constant Velocity (1D Points)

- State vector: position $p$ and velocity $v$

$$x_t = \begin{bmatrix} p_t \\ v_t \end{bmatrix} \qquad \begin{aligned} p_t &= p_{t-1} + (\Delta t) v_{t-1} + \varepsilon \\ v_t &= v_{t-1} + \xi \end{aligned}$$

(greek letters denote noise terms)

$$x_t = D_t x_{t-1} + noise = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p_{t-1} \\ v_{t-1} \end{bmatrix} + noise$$

- Measurement is position only

$$y_t = M x_t + noise = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} p_t \\ v_t \end{bmatrix} + noise$$



Slide credit: Svetlana Lazebnik, Kristen Grauman

# Recap: Constant Acceleration (1D Points)

- State vector: position $p$, velocity $v$, and acceleration $a$.

$$x_t = \begin{bmatrix} p_t \\ v_t \\ a_t \end{bmatrix} \qquad \begin{aligned} p_t &= p_{t-1} + (\Delta t) v_{t-1} + \tfrac{1}{2}(\Delta t)^2 a_{t-1} + \varepsilon \\ v_t &= v_{t-1} + (\Delta t) a_{t-1} + \xi \\ a_t &= a_{t-1} + \zeta \end{aligned}$$

(greek letters denote noise terms)

$$x_t = D_t x_{t-1} + noise = \begin{bmatrix} 1 & \Delta t & \frac{1}{2}(\Delta t)^2 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_{t-1} \\ v_{t-1} \\ a_{t-1} \end{bmatrix} + noise$$

- Measurement is position only

$$y_t = M x_t + noise = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_t \\ v_t \\ a_t \end{bmatrix} + noise$$



Slide credit: Svetlana Lazebnik, Kristen Grauman

# Recap: General Motion Models

- Assuming we have differential equations for the motion
  - E.g. for (undampened) periodic motion of a linear spring

$$\frac{d^2 p}{dt^2} = -p$$

- Substitute variables to transform this into linear system

$$p_1 = p \qquad p_2 = \frac{dp}{dt} \qquad p_3 = \frac{d^2 p}{dt^2}$$

- Then we have

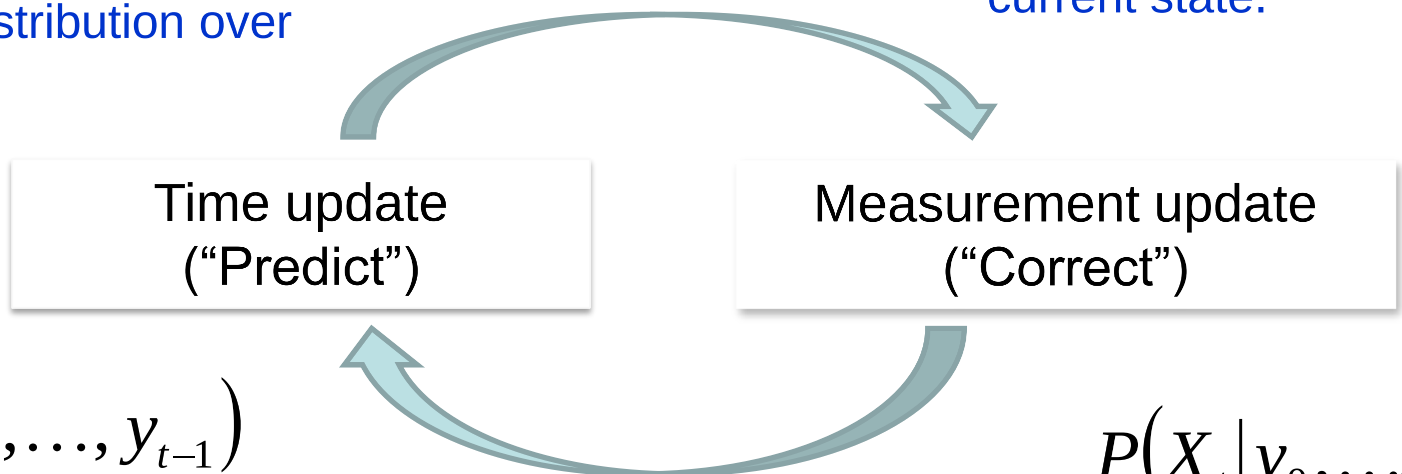
$$x_t = \begin{bmatrix} p_{1,t} \\ p_{2,t} \\ p_{3,t} \end{bmatrix} \qquad \begin{aligned} p_{1,t} &= p_{1,t-1} + (\Delta t) p_{2,t-1} + \tfrac{1}{2}(\Delta t)^2 p_{3,t-1} + \varepsilon \\ p_{2,t} &= p_{2,t-1} + (\Delta t) p_{3,t-1} + \xi \\ p_{3,t} &= -p_{1,t-1} + \zeta \end{aligned} \qquad D_t = \begin{bmatrix} 1 & \Delta t & \frac{1}{2}(\Delta t)^2 \\ 0 & 1 & \Delta t \\ -1 & 0 & 0 \end{bmatrix}$$

# Recap: The Kalman Filter

Know corrected state from previous time step, and all measurements up to the current one
→ Predict distribution over next state.

*Receive measurement*

Know prediction of state, and next measurement
→Update distribution over current state.




$$P\left(X_t \mid y_0, \ldots, y_{t-1}\right)$$

$$P\left(X_t \mid y_0, \ldots, y_t\right)$$

Mean and std. dev. of predicted state:

$$\mu_t^-, \sigma_t^-$$

*Time advances: t++*

Mean and std. dev. of corrected state:

$$\mu_t^+, \sigma_t^+$$



Slide credit: Kristen Grauman

# Recap: General Kalman Filter (>1dim)

**PREDICT**

$$x_t^- = D_t x_{t-1}^+$$

$$\Sigma_t^- = D_t \Sigma_{t-1}^+ D_t^T + \Sigma_{d_t}$$

**CORRECT**

$$K_t = \Sigma_t^- M_t^T \left( M_t \Sigma_t^- M_t^T + \Sigma_{m_t} \right)^{-1}$$

$$x_t^+ = x_t^- + K_t \left( y_t - M_t x_t^- \right)$$

"residual"

"Kalman gain"

$$\Sigma_t^+ = \left( I - K_t M_t \right) \Sigma_t^-$$

More weight on residual when measurement error covariance approaches 0.

Less weight on residual as a priori estimate error covariance approaches 0.

for derivations, see F&P Chapter 17.3

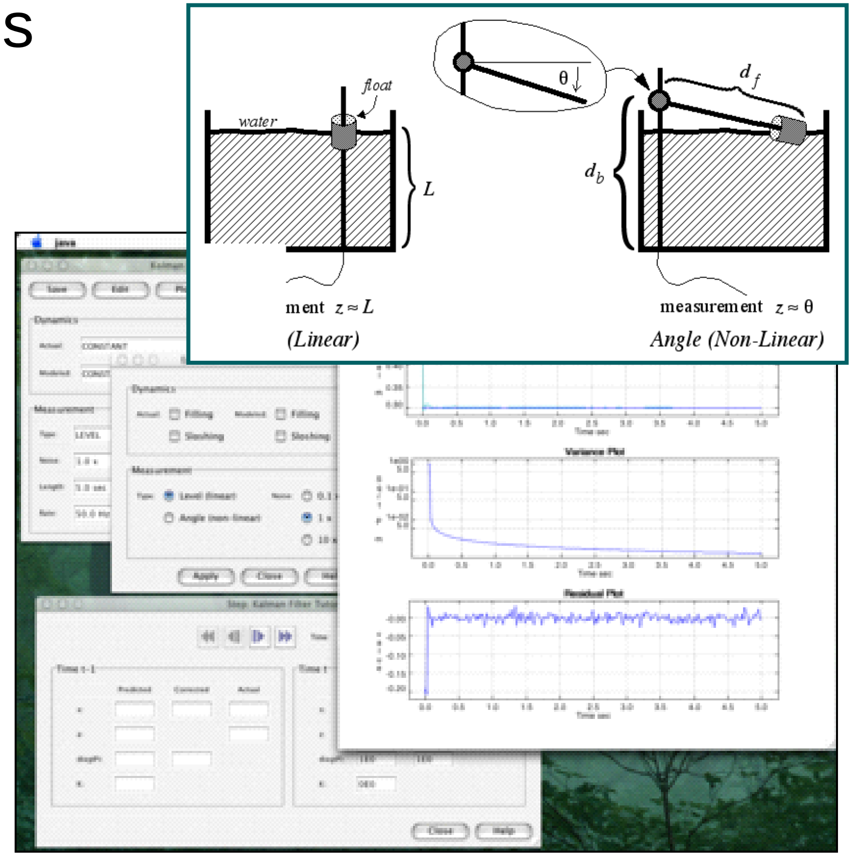


Slide credit: Kristen Grauman

# Resources: Kalman Filter Web Site

http://www.cs.unc.edu/~welch/kalman

- Electronic and printed references
  - Book lists and recommendations
  - Research papers
  - Links to other sites
  - Some software
- News
- Java-Based KF Learning Tool
  - On-line 1D simulation
  - Linear and non-linear
  - Variable dynamics






Slide adapted from Greg Welch

# Remarks

- Try it!
  - Not too hard to understand or program
- Start simple
  - Experiment in 1D
  - Make your own filter in Matlab, etc.
- Note: the Kalman filter “wants to work”
  - Debugging can be difficult
  - Errors can go un-noticed



Slide adapted from Greg Welch

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  - Basic ideas
  - Limitations
  - Extensions
- Particle Filters
  - Basic ideas
  - Propagation of general densities
  - Factored sampling
- Case study
  - Detector Confidence Particle Filter
  - Role of the different elements

# Extension: Extended Kalman Filter (EKF)

- Basic idea
  - State transition and observation model don't need to be linear functions of the state, but just need to be differentiable.

$$x_t = g(x_{t-1}, u_t) + \varepsilon$$

$$y_t = h(x_t) + \delta$$

  - The EKF essentially linearizes the nonlinearity around the current estimate by a Taylor expansion.
- Properties
  - Unlike the linear KF, the EKF is in general *not* an optimal estimator.
    - If the initial estimate is wrong, the filter may quickly diverge.
  - Still, it's the de-facto standard in many applications
    - Including navigation systems and GPS

# Recap: Kalman Filter – Detailed Algorithm

- Algorithm summary
  - Assumption: linear model

$$
\begin{aligned}
\mathbf{x}_t &= \mathbf{D}_t \mathbf{x}_{t-1} + \varepsilon_t \\
\mathbf{y}_t &= \mathbf{M}_t \mathbf{x}_t + \delta_t
\end{aligned}
$$

  - Prediction step

$$
\begin{aligned}
\mathbf{x}_t^- &= \mathbf{D}_t \mathbf{x}_{t-1}^+ \\
\mathbf{\Sigma}_t^- &= \mathbf{D}_t \mathbf{\Sigma}_{t-1}^+ \mathbf{D}_t^T + \mathbf{\Sigma}_{d_t}
\end{aligned}
$$

  - Correction step

$$
\begin{aligned}
\mathbf{K}_t &= \mathbf{\Sigma}_t^- \mathbf{M}_t^T \left( \mathbf{M}_t \mathbf{\Sigma}_t^- \mathbf{M}_t^T + \mathbf{\Sigma}_{m_t} \right)^{-1} \\
\mathbf{x}_t^+ &= \mathbf{x}_t^- + \mathbf{K}_t \left( \mathbf{y}_t - \mathbf{M}_t \mathbf{x}_t^- \right) \\
\mathbf{\Sigma}_t^+ &= \left( \mathbf{I} - \mathbf{K}_t \mathbf{M}_t \right) \mathbf{\Sigma}_t^-
\end{aligned}
$$

# Extended Kalman Filter (EKF)

- Algorithm summary
  - Nonlinear model

$$\mathbf{x}_t = \mathbf{g}(\mathbf{x}_{t-1}) + \varepsilon_t$$
$$\mathbf{y}_t = \mathbf{h}(\mathbf{x}_t) + \delta_t$$

with the Jacobians

  - Prediction step

$$\mathbf{x}_t^- = \mathbf{g}(\mathbf{x}_{t-1}^+)$$
$$\mathbf{\Sigma}_t^- = \mathbf{G}_t \mathbf{\Sigma}_{t-1}^+ \mathbf{G}_t^T + \mathbf{\Sigma}_{d_t}$$

$$\mathbf{G}_t = \left.\frac{\partial \mathbf{g}(\mathbf{x})}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}_{t-1}^+}$$

  - Correction step

$$\mathbf{K}_t = \mathbf{\Sigma}_t^- \mathbf{H}_t^T \left(\mathbf{H}_t \mathbf{\Sigma}_t^- \mathbf{H}_t^T + \mathbf{\Sigma}_{m_t}\right)^{-1}$$
$$\mathbf{x}_t^+ = \mathbf{x}_t^- + \mathbf{K}_t \left(\mathbf{y}_t - \mathbf{h}(\mathbf{x}_t^-)\right)$$
$$\mathbf{\Sigma}_t^+ = (\mathbf{I} - \mathbf{K}_t \mathbf{H}_t)\, \mathbf{\Sigma}_t^-$$

$$\mathbf{H}_t = \left.\frac{\partial \mathbf{h}(\mathbf{x})}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}_t^-}$$

# Kalman Filter – Other Extensions

- Unscented Kalman Filter (UKF)
  - Used for models with highly nonlinear predict and update functions.
  - Here, the EKF can give very poor performance, since the covariance is propagated through linearization of the non-linear model.
  - Idea (UKF): Propagate just a few sample points ("*sigma points*") around the mean exactly, then recover the covariance from them.
  - More accurate results than the EKF's Taylor expansion approximation.
- Ensemble Kalman Filter (EnKF)
  - Represents the distribution of the system state using a collection (an *ensemble*) of state vectors.
  - Replace covariance matrix by *sample covariance* from ensemble.
  - Still basic assumption that all prob. distributions involved are Gaussian.
  - EnKFs are especially suitable for problems with a large number of variables.

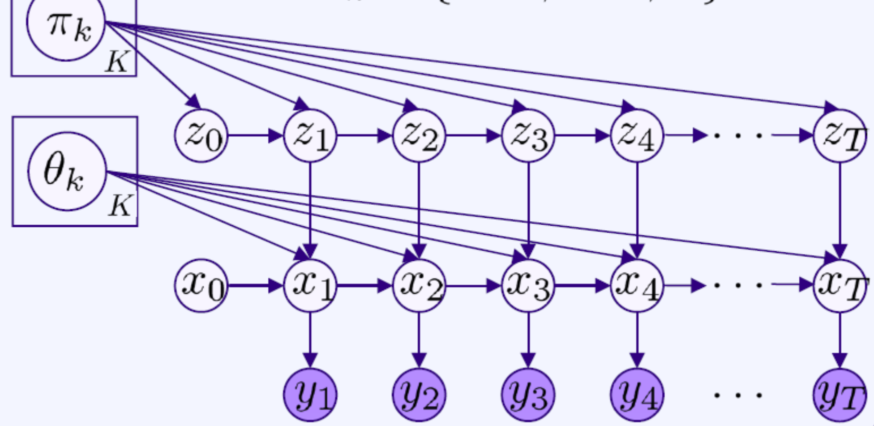
# Even More Extensions

Switching linear dynamical system (SLDS):

$$\theta_k = \{A^{(k)}, \Sigma^{(k)}, R\}$$

$$z_t \sim \pi_{z_{t-1}}$$
$$x_t = A^{(z_t)} x_{t-1} + e_t(z_t)$$
$$y_t = C x_t + w_t$$

$$e_t \sim \mathcal{N}(0, \Sigma^{(z_t)}) \quad w_t \sim \mathcal{N}(0, R)$$




- Switching Linear Dynamic System (SLDS)
  - Use a set of $k$ dynamic models $A^{(1)}$,...,$A^{(k)}$, each of which describes a different dynamic behavior.
  - Hidden variable $z_t$ determines which model is active at time $t$.
  - A switching process can change $z_t$ according to distribution$\pi_{z_{t-1}}$.

Figure source: Erik Sudderth

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- Particle Filters
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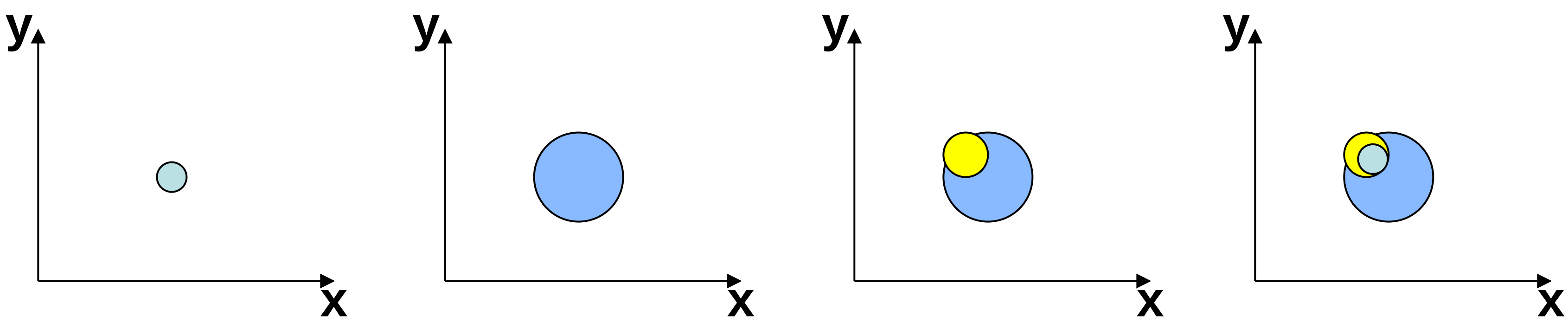
  - Propagation of general densities
  - Factored sampling
- Case study
  - Detector Confidence Particle Filter
  - Role of the different elements

Today: only main ideas

Formal introduction next lecture

# When Is A Single Hypothesis Too Limiting?

Initial position | Prediction | Measurement | Update




- Consider this example: say we are tracking the face on the right using a skin color blob to get our measurement.


Video from Jojic & Frey

Figure from Thrun & Kosecka

Slide credit: Kristen Grauman

# Propagation of General Densities

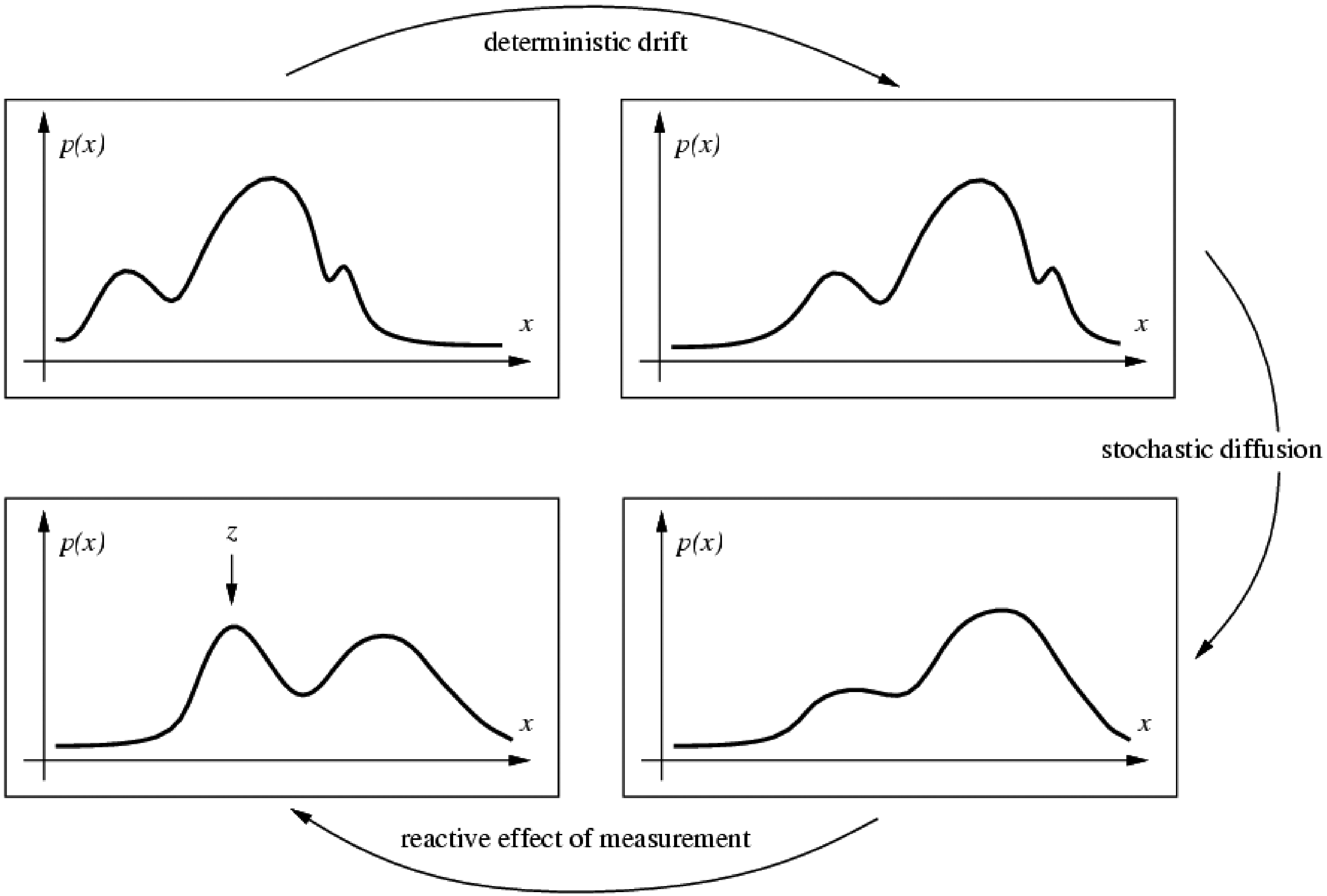
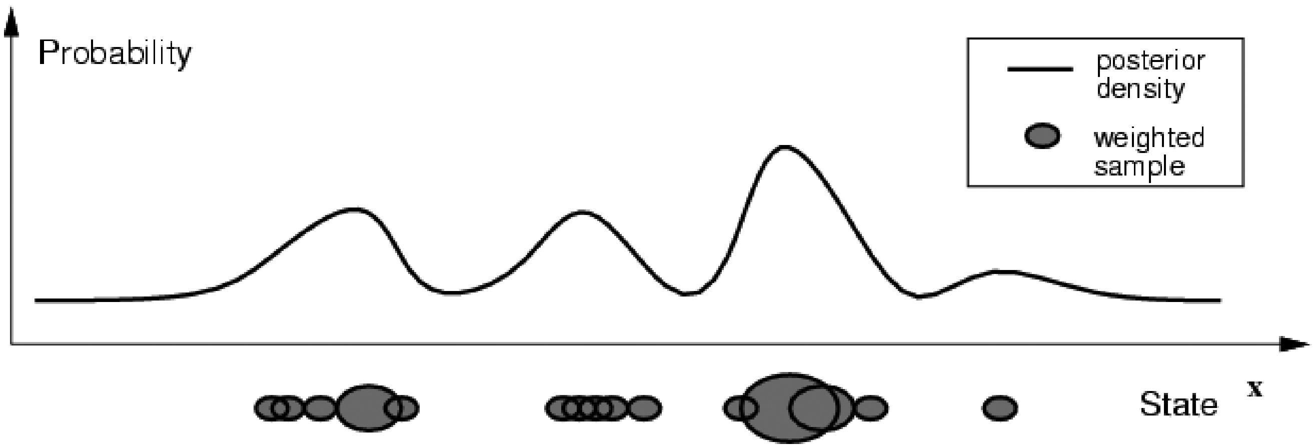



Figure from Isard & Blake

# Factored Sampling




- Idea: Represent state distribution non-parametrically
  - Prediction: Sample points from prior density for the state, $P(X)$
  - Correction: Weight the samples according to $P(Y|X)$

$$P(X_t \mid y_0, \ldots, y_t) = \frac{P(y_t \mid X_t) P(X_t \mid y_0, \ldots, y_{t-1})}{\int P(y_t \mid X_t) P(X_t \mid y_0, \ldots, y_{t-1}) dX_t}$$



Slide credit: Svetlana Lazebnik

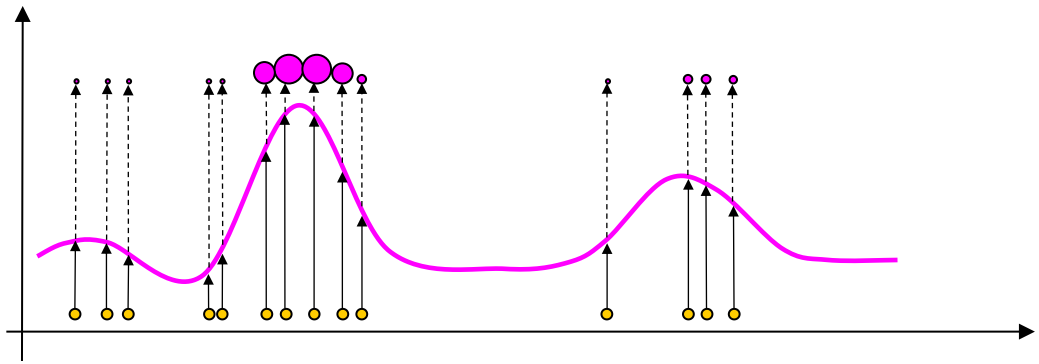
# Particle Filtering

- (Also known as Sequential Monte Carlo Methods)

- Idea
  - We want to use sampling to propagate densities over time (i.e., across frames in a video sequence).
  - At each time step, represent posterior $P(X_t|Y_t)$ with weighted sample set.
  - Previous time step's sample set $P(X_{t-1}|Y_{t-1})$ is passed to next time step as the effective prior.

# Particle Filtering

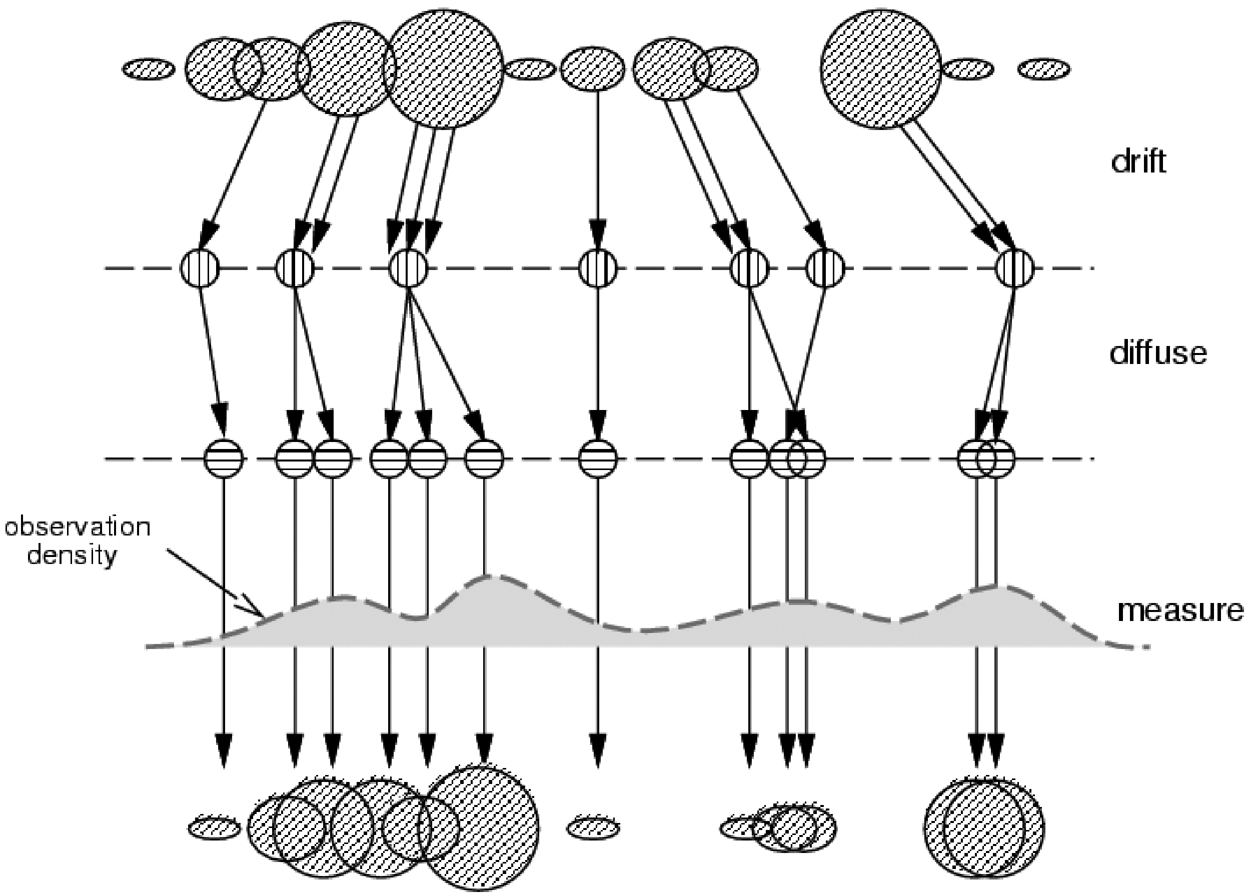
- Many variations, one general concept:
  - *Represent the posterior pdf by a set of randomly chosen weighted samples (particles)*




  - Randomly Chosen = Monte Carlo (MC)
  - As the number of samples become very large – the characterization becomes an equivalent representation of the true pdf.


Slide adapted from Michael Rubinstein

# Particle Filtering




- Start with weighted samples from previous time step
- Sample and shift according to dynamics model
- Spread due to randomness; this is predicted density $P(X_t|Y_{t-1})$
- Weight the samples according to observation density
- Arrive at corrected density estimate $P(X_t|Y_t)$

M. Isard and A. Blake, CONDENSATION -- conditional density propagation for visual tracking, IJCV 29(1):5-28, 1998

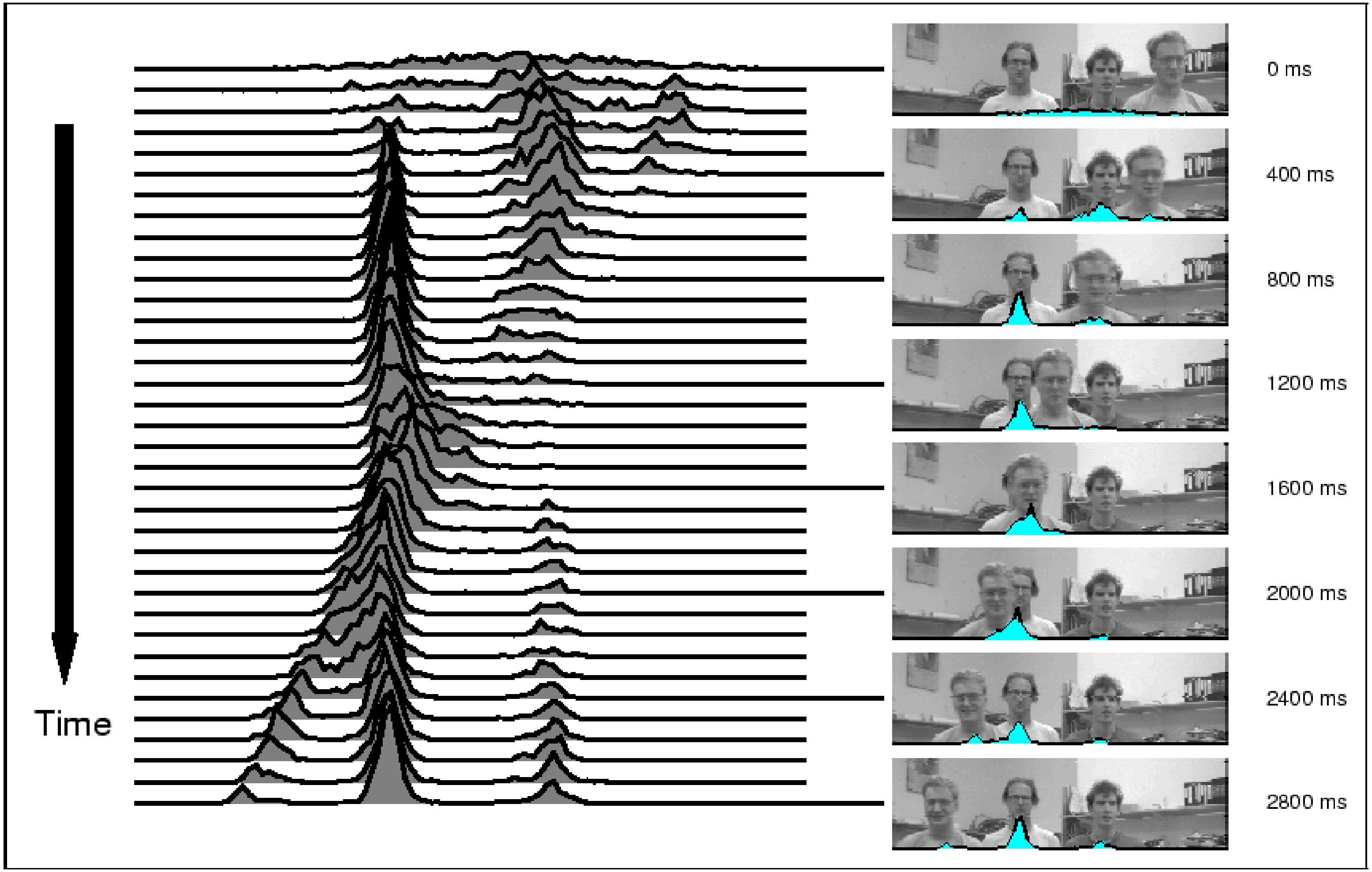


Slide credit: Svetlana Lazebnik

# Particle Filtering – Visualization

Code and video available from
http://www.robots.ox.ac.uk/~misard/condensation.html

# Particle Filtering Results




http://www.robots.ox.ac.uk/~misard/condensation.html

# Particle Filtering Results

- Some more examples

http://www.robots.ox.ac.uk/~misard/condensation.html



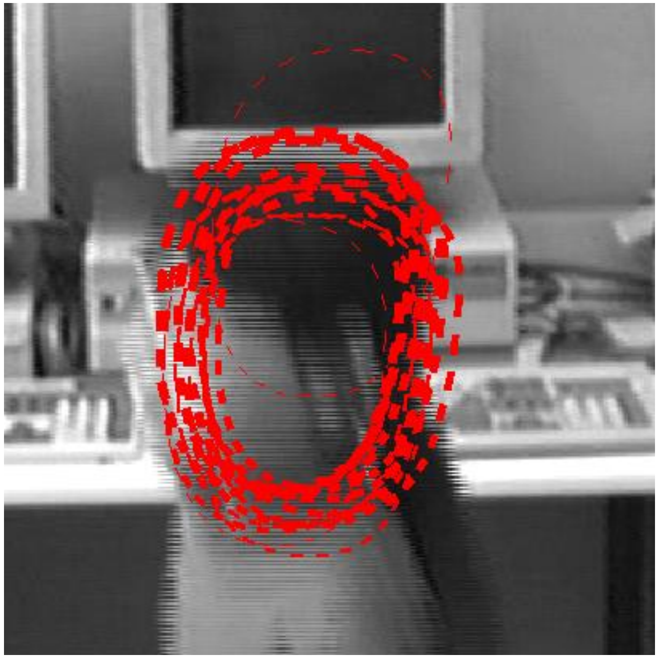


Videos from Isard & Blake

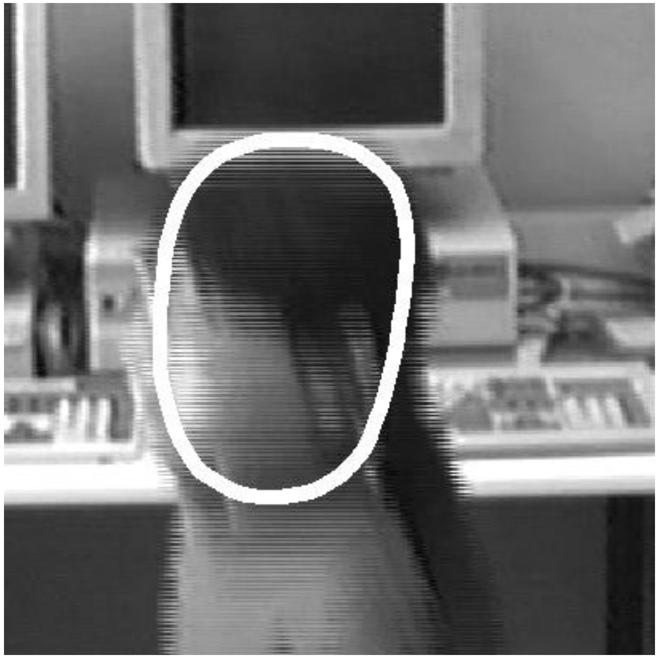
# Obtaining a State Estimate

- Note that there's no explicit state estimate maintained, just a "cloud" of particles
- Can obtain an estimate at a particular time by querying the current particle set
- Some approaches
  - "Mean" particle
    - Weighted sum of particles
    - Confidence: inverse variance
  - Really want a mode finder—mean of tallest peak

# Condensation: Estimating Target State



State samples
(thickness proportional to weight)



From Isard & Blake, 1998

Mean of weighted
state samples



Slide credit: Marc Pollefeys

Figures from Isard & Blake

# Summary: Particle Filtering

- <u>Pros:</u>
  - Able to represent arbitrary densities
  - Converging to true posterior even for non-Gaussian and nonlinear system
  - Efficient: particles tend to focus on regions with high probability
  - Works with many different state spaces
    - E.g. articulated tracking in complicated joint angle spaces
  - Many extensions available

# Summary: Particle Filtering

- <u>Cons / Caveats:</u>
  - #Particles is important performance factor
    - Want as few particles as possible for efficiency.
    - But need to cover state space sufficiently well.
  - Worst-case complexity grows exponentially in the dimensions
  - Multimodal densities possible, but still single object
    - Interactions between multiple objects require special treatment.
    - Not handled well in the particle filtering framework (state space explosion).

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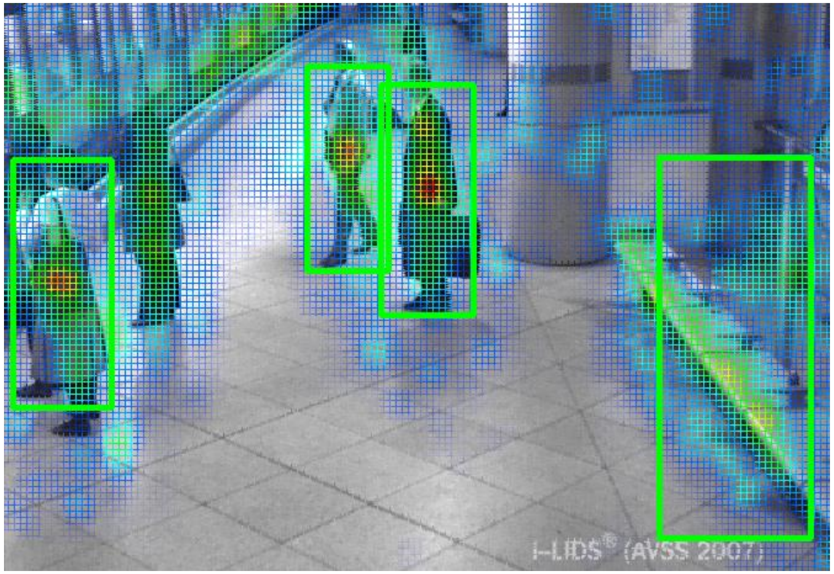
# Challenge: Unreliable Object Detectors

- Example:
  - Low-res webcam footage (320×240), MPEG compressed

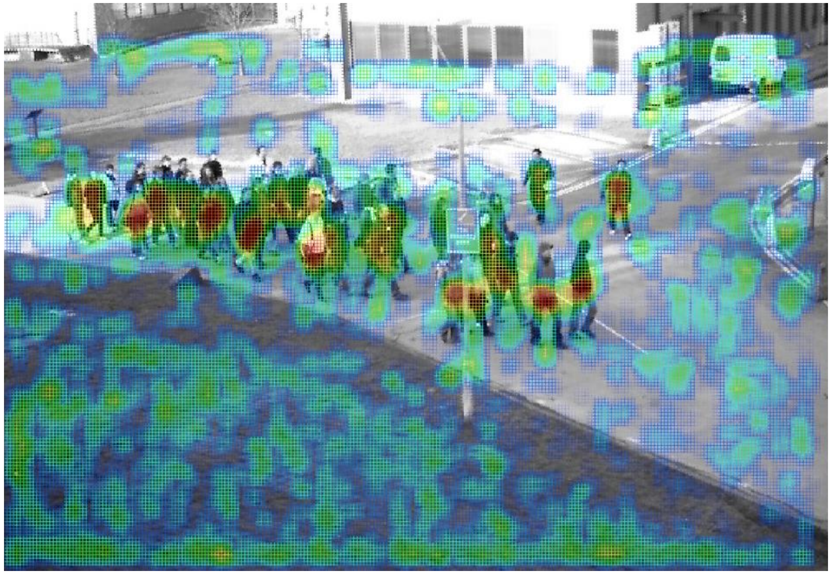
Detector input | Tracker output

How to get from here… ? …to here?

# Tracking based on Detector Confidence



(using ISM detector)



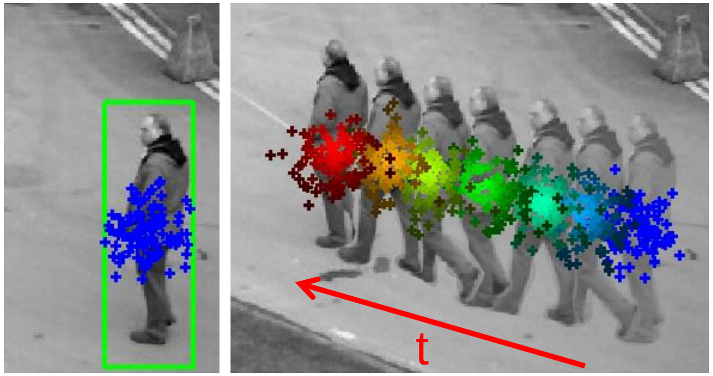
(using HOG detector)

- Detector output is often not perfect
  - Missing detections and false positives
  - But continuous confidence still contains useful cues.

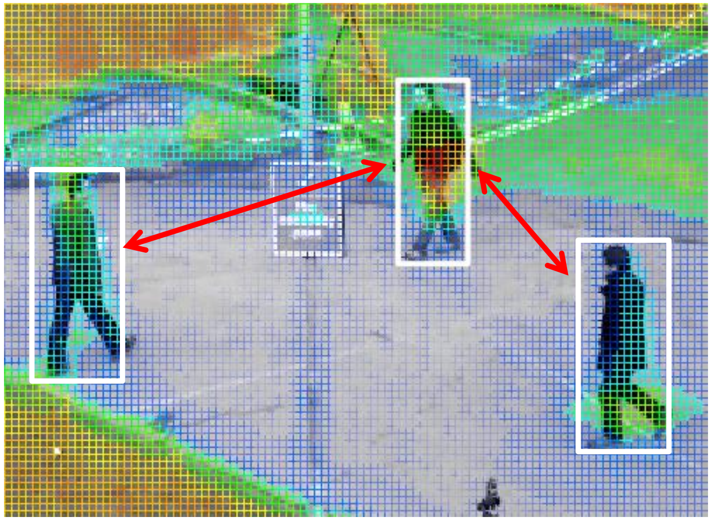
- Idea pursued here:
  - Use continuous detector confidence to track persons over time.

# Main Ideas

- Detector confidence particle filter
  - Initialize particle cloud on strong object detections.
  - Propagate particles using continuous detector confidence as observation model.



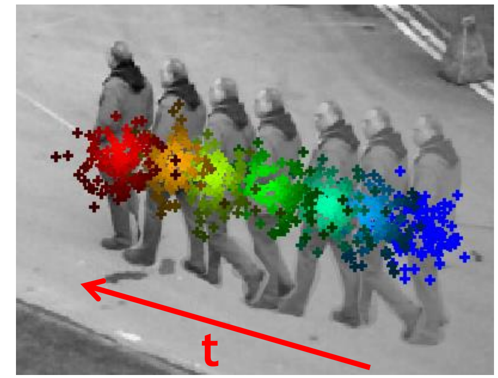

- Disambiguate between different persons
  - Train a person-specific classifier with online boosting.
  - Use classifier output to distinguish between nearby persons.



[Breitenstein, Reichlin, Leibe et al., ICCV'09]

# Detector Confidence Particle Filter

- State: $\mathbf{x} = \{x, y, u, v\}$




- Motion model (constant velocity)

$$\begin{aligned}(x,y)_t &= (x,y)_{t-1} + (u,v)_{t-1} \cdot \Delta t + \varepsilon_{(x,y)} \\ (u,v)_t &= (u,v)_{t-1} + \varepsilon_{(u,v)}\end{aligned}$$

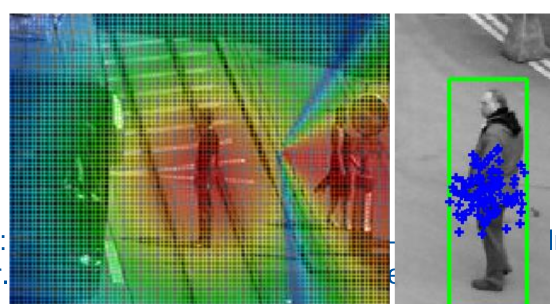
- Observation model

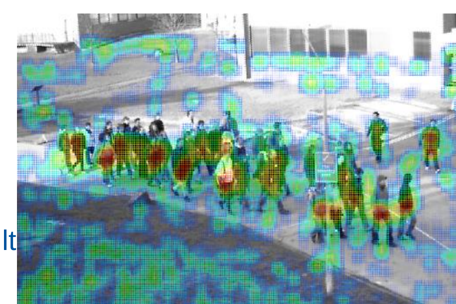
$$w_{tr,p} = p(y_t | x_t^{(i)}) = \boxed{\beta \cdot \mathcal{I}(tr) \cdot p_{\mathcal{N}}(p - d^*)} + \boxed{\gamma \cdot d_c(p) \cdot p_o(tr)} + \boxed{\eta \cdot c_{tr}(p)}$$

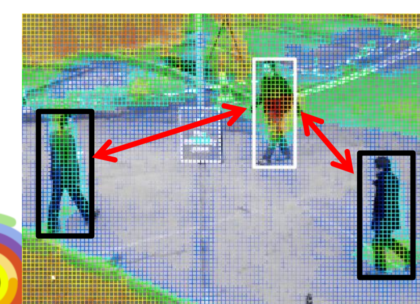
Discrete detections

Detector confidence

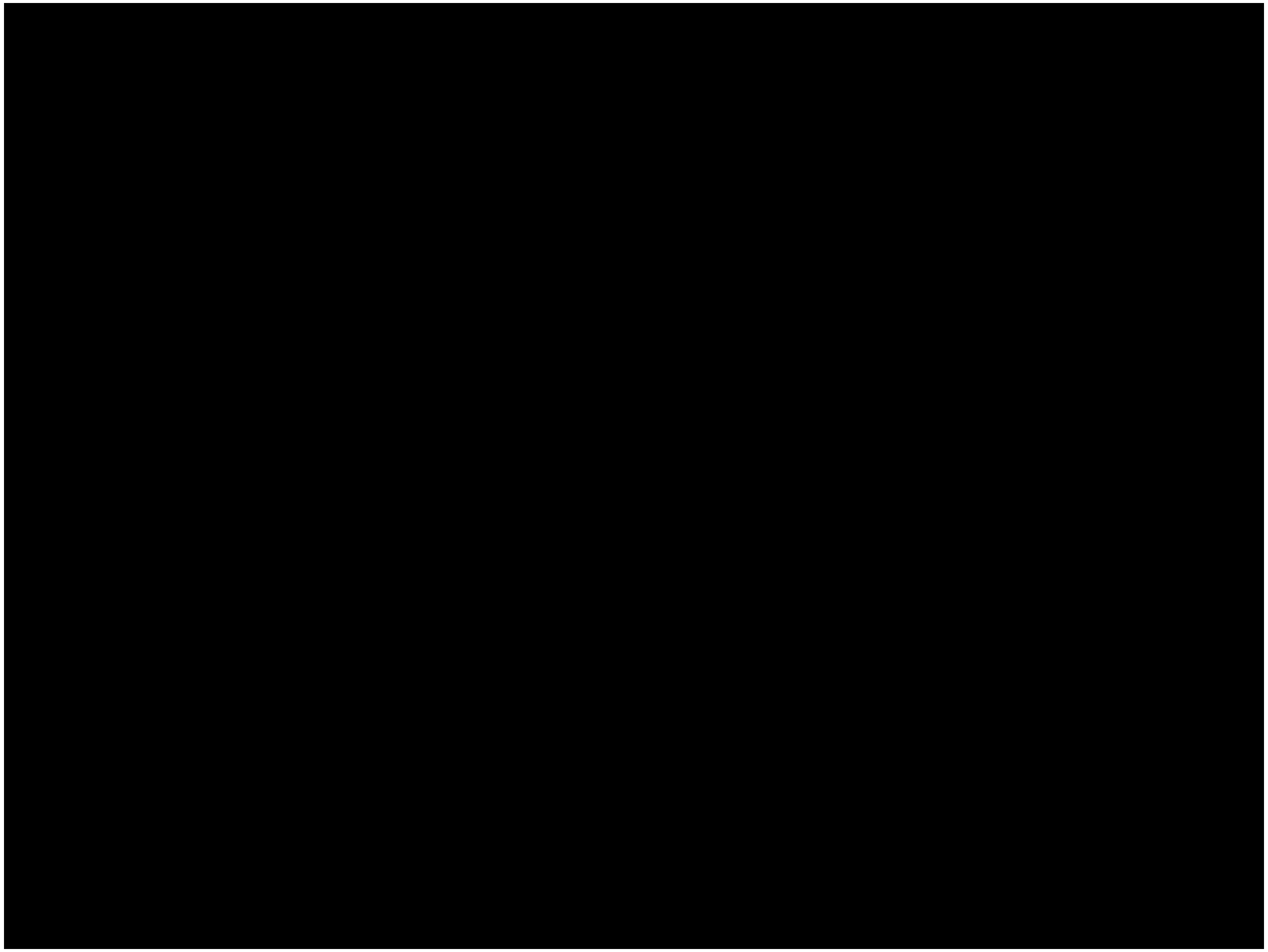
Classifier confidence







# When Is Which Term Useful?

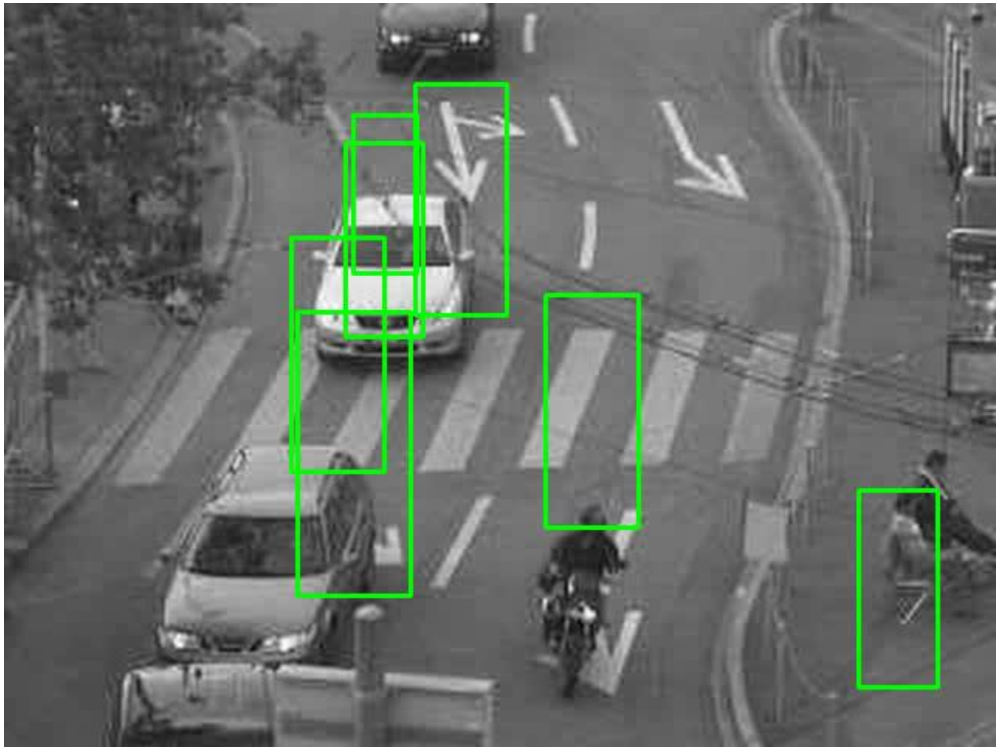


**Discrete detections** **Detector confidence** **Classifier confidence**

# Each Observation Term Increases Robustness!

| Observation Model Terms | MOTP | MOTA | FN | FP | ID Sw. |
|---|---|---|---|---|---|
| 1: Det+DetConf+Class | 70.0% | 72.9% | 26.8% | 0.3% | 0 |
| 2: Det+DetConf | 64.0% | 54.5% | 28.2% | 17.2% | 5 |
| 3: Det+Class | 65.0% | 55.3% | 31.3% | 13.4% | 0 |
| 4: Det | 67.0% | 40.9% | 30.7% | 28.0% | 10 |

Detector only

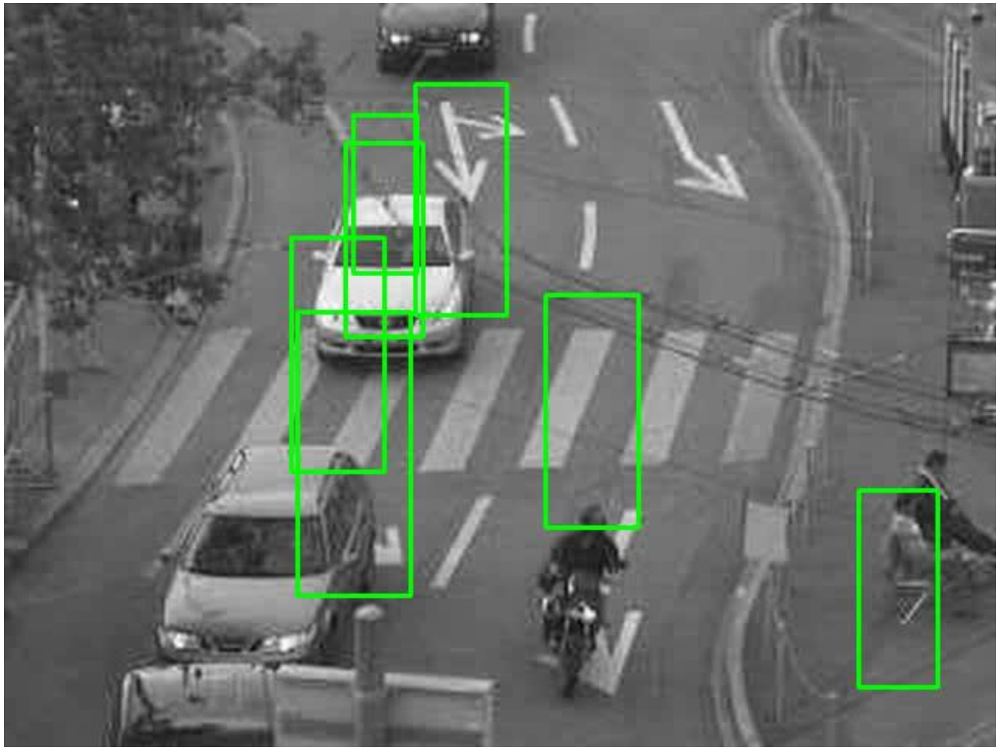


CLEAR MOT scores

# Each Observation Term Increases Robustness!

| Observation Model Terms | MOTP | MOTA | FN | FP | ID Sw. |
|---|---|---|---|---|---|
| 1: Det+DetConf+Class | 70.0% | 72.9% | 26.8% | 0.3% | 0 |
| 2: Det+DetConf | 64.0% | 54.5% | 28.2% | 17.2% | 5 |
| 3: Det+Class | 65.0% | 55.3% | 31.3% | 13.4% | 0 |
| 4: Det | 67.0% | 40.9% | 30.7% | 28.0% | 10 |

Detector
+ Confidence



CLEAR MOT scores



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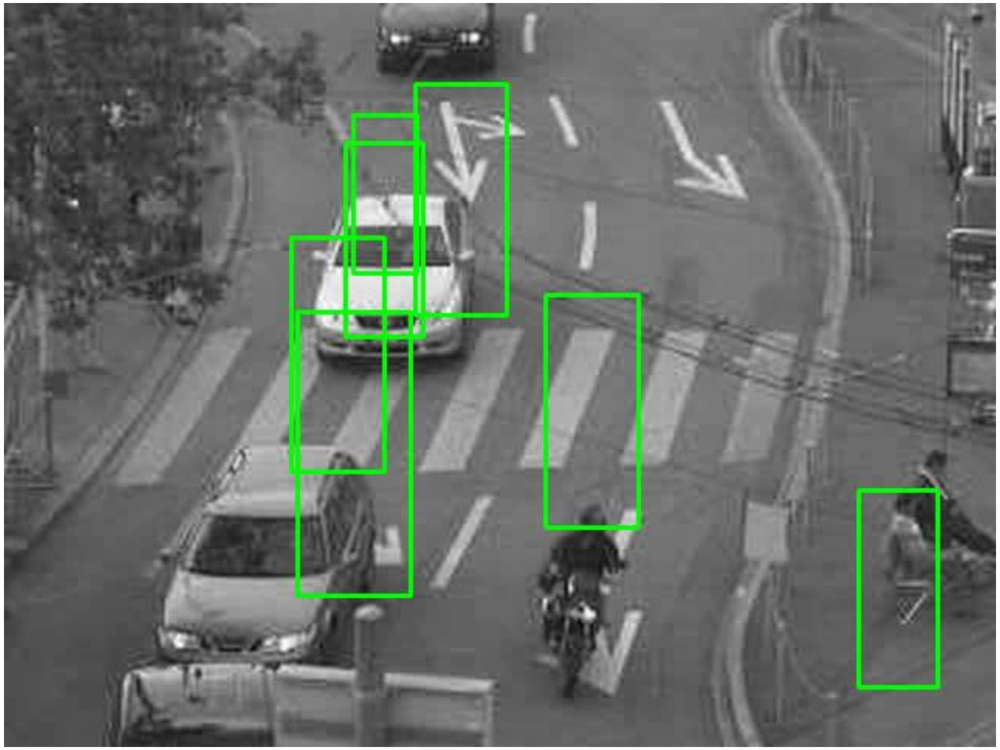
# Each Observation Term Increases Robustness!

| Observation Model Terms | MOTP | MOTA | FN | FP | ID Sw. |
|---|---|---|---|---|---|
| 1: Det+DetConf+Class | 70.0% | 72.9% | 26.8% | 0.3% | 0 |
| 2: Det+DetConf | 64.0% | 54.5% | 28.2% | 17.2% | 5 |
| 3: Det+Class | 65.0% | 55.3% | 31.3% | 13.4% | 0 |
| 4: Det | 67.0% | 40.9% | 30.7% | 28.0% | 10 |

Detector
+ Classifier



CLEAR MOT scores

# Each Observation Term Increases Robustness!

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| 1: Det+DetConf+Class | 70.0% | 72.9% | 26.8% | 0.3% | 0 |
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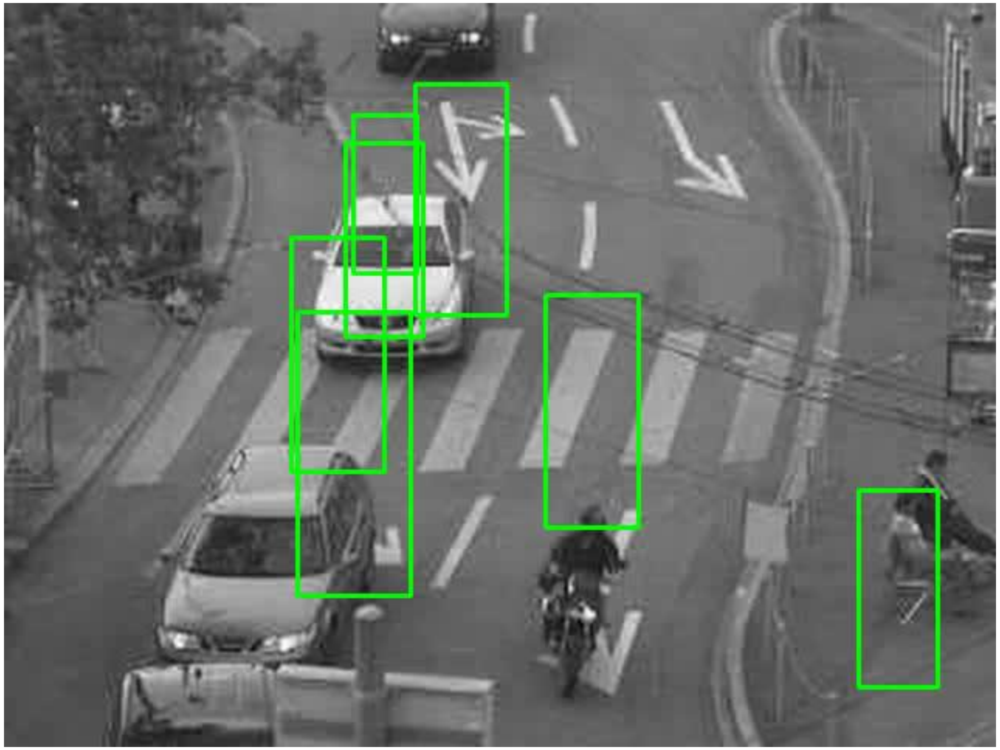
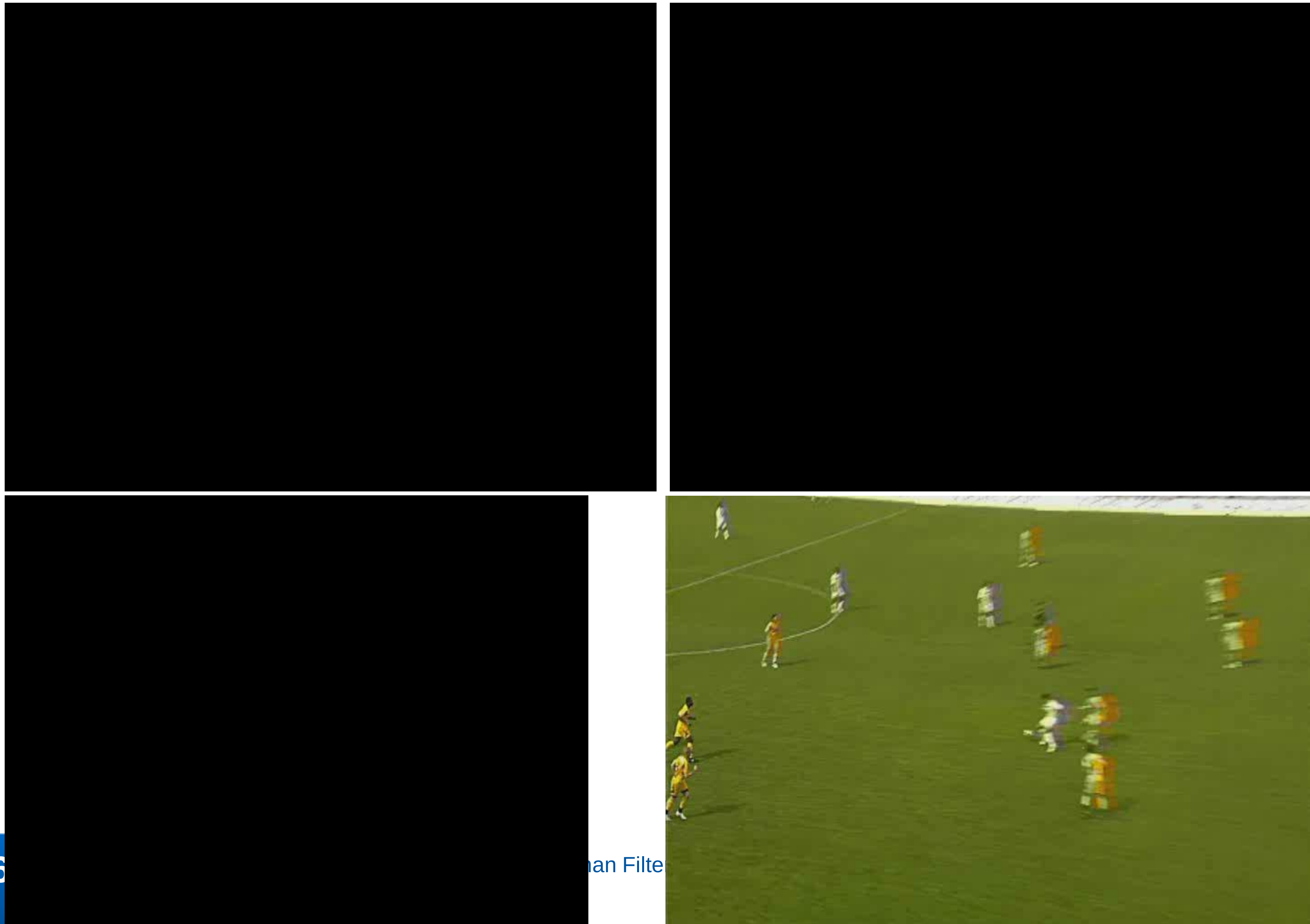
Detector
+ Confidence
+ Classifier



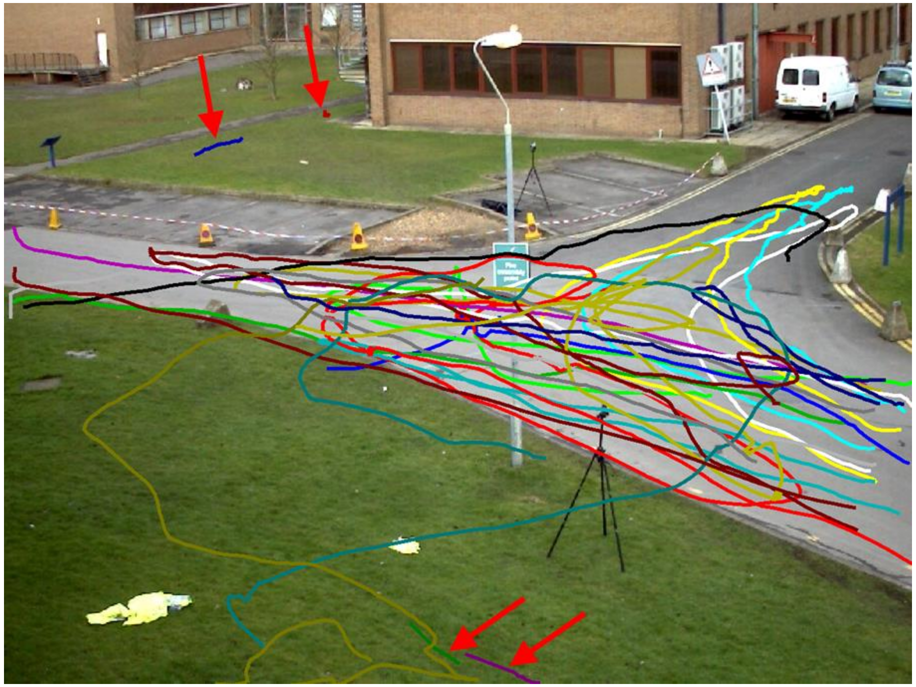
False negatives, false positives, and ID switches decrease!

CLEAR MOT scores

# Qualitative Results



# Remaining Issues



- Some false positive initializations at wrong scales…
  - Due to limited scale range of the person detector.
  - Due to boundary effects of the person detector.

# References and Further Reading

- A good tutorial on Particle Filters
  - M.S. Arulampalam, S. Maskell, N. Gordon, T. Clapp. A Tutorial on Particle Filters for Online Nonlinear/Non-Gaussian Bayesian Tracking. In *IEEE Transactions on Signal Processing*, Vol. 50(2), pp. 174-188, 2002.

- The CONDENSATION paper
  - M. Isard and A. Blake, CONDENSATION - conditional density propagation for visual tracking, IJCV 29(1):5-28, 1998