# Computer Vision – Lecture 3

## Gradients & Edges

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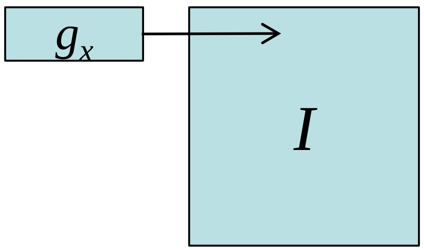
# Course Outline

- Image Processing Basics
  - Image Formation
  - Binary Image Processing
  - Linear Filters
  - Edge & Structure Extraction
- Segmentation
- Local Features & Matching
- Object Recognition and Categorization
- Deep Learning
- 3D Reconstruction

# Topics of This Lecture

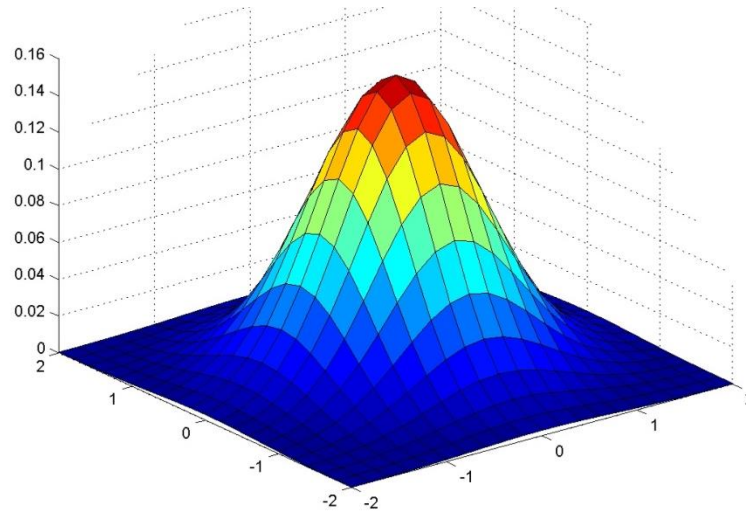
- Recap: Linear Filters
- Multi-Scale representations
  - How to properly rescale an image?
- Filters as templates
  - Correlation as template matching
- Image gradients
  - Derivatives of Gaussian
- Edge detection
  - Canny edge detector



# Recap: Gaussian Smoothing

- Gaussian kernel

$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$




- Rotationally symmetric
- Weights nearby pixels more than distant ones
  - This makes sense as 'probabilistic' inference about the signal
- A Gaussian gives a good model of a fuzzy blob

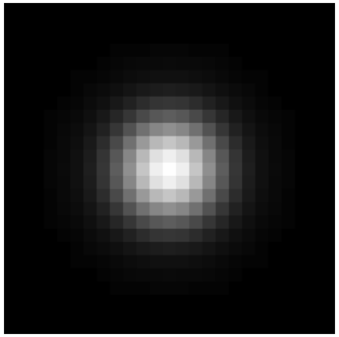
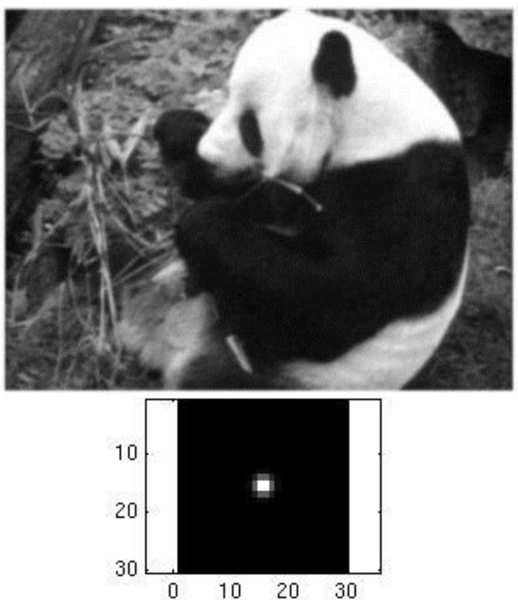
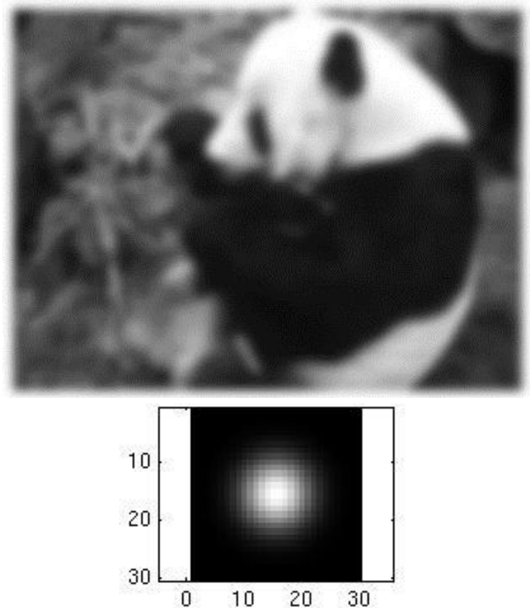



Image Source: Forsyth & Ponce

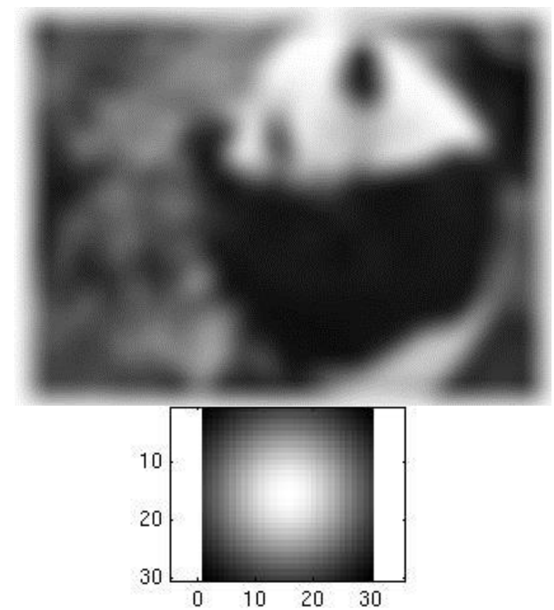
# Recap: Smoothing with a Gaussian

- Parameter σ is the “scale” / “width” / “spread” of the Gaussian kernel and controls the amount of smoothing.




…

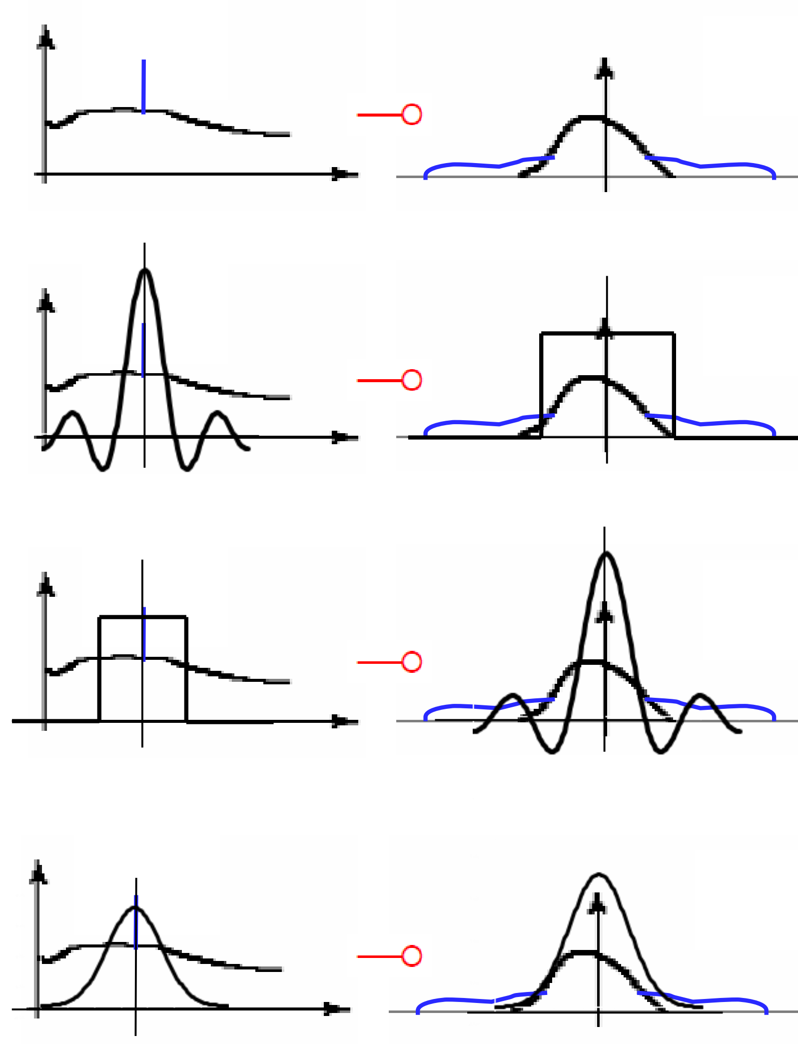



```
for sigma=1:3:10
   h = fspecial('gaussian`, fsize, sigma);
   out = imfilter(im, h);
   imshow(out);
   pause;
end
```

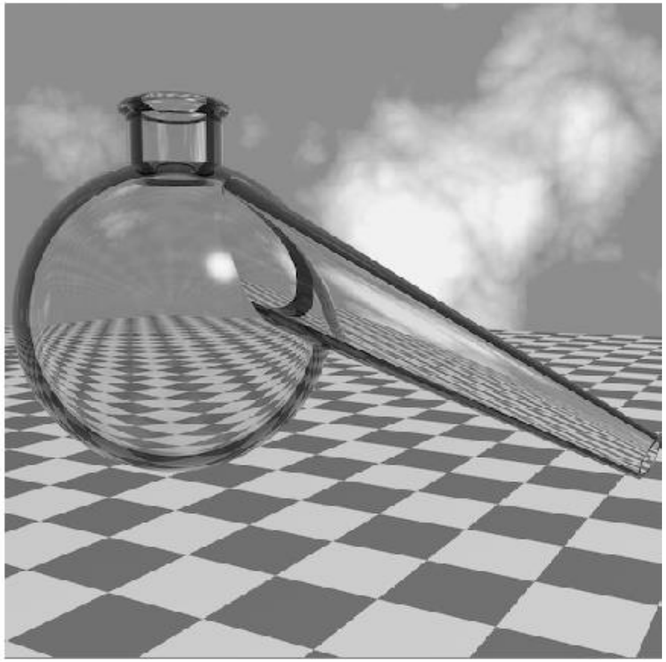
Slide credit: Kristen Grauman

# Recap: Effect of Filtering

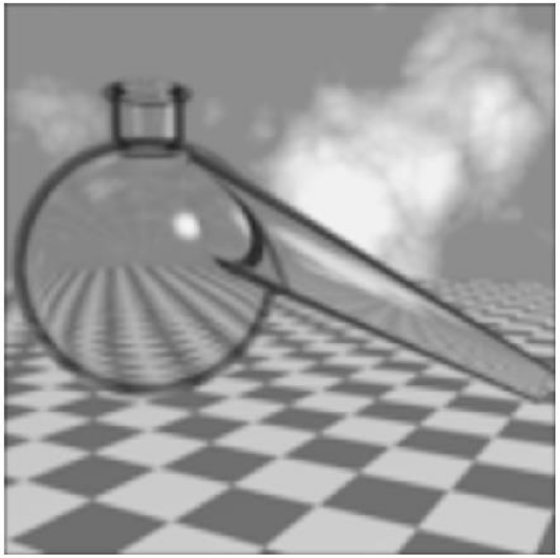
- Noise introduces high frequencies. To remove them, we want to apply a "low-pass" filter.
- The ideal filter shape in the frequency domain would be a box. But this transfers to a spatial sinc, which has infinite spatial support.
- A compact spatial box filter transfers to a frequency sinc, which creates artifacts.
- A Gaussian has compact support in both domains. This makes it a convenient choice for a low-pass filter.



# Recap: Low-Pass vs. High-Pass



Original image



Low-pass filtered

High-pass filtered



Image Source: S. Chenney

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- Filters as templates
  - Correlation as template matching
- Image gradients
  - Derivatives of Gaussian
- Edge detection
  - Canny edge detector

# Motivation: Fast Search Across Scales

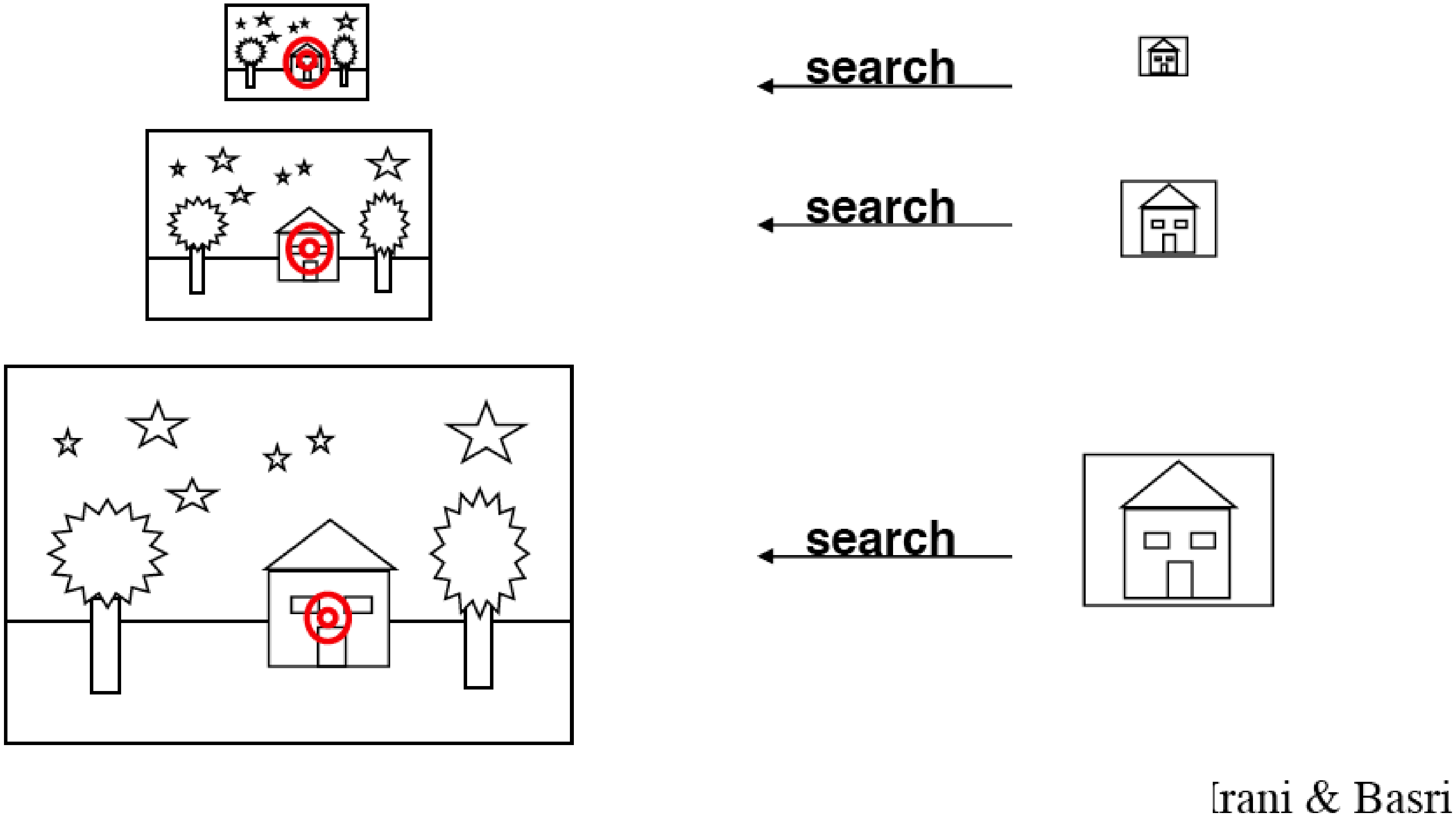
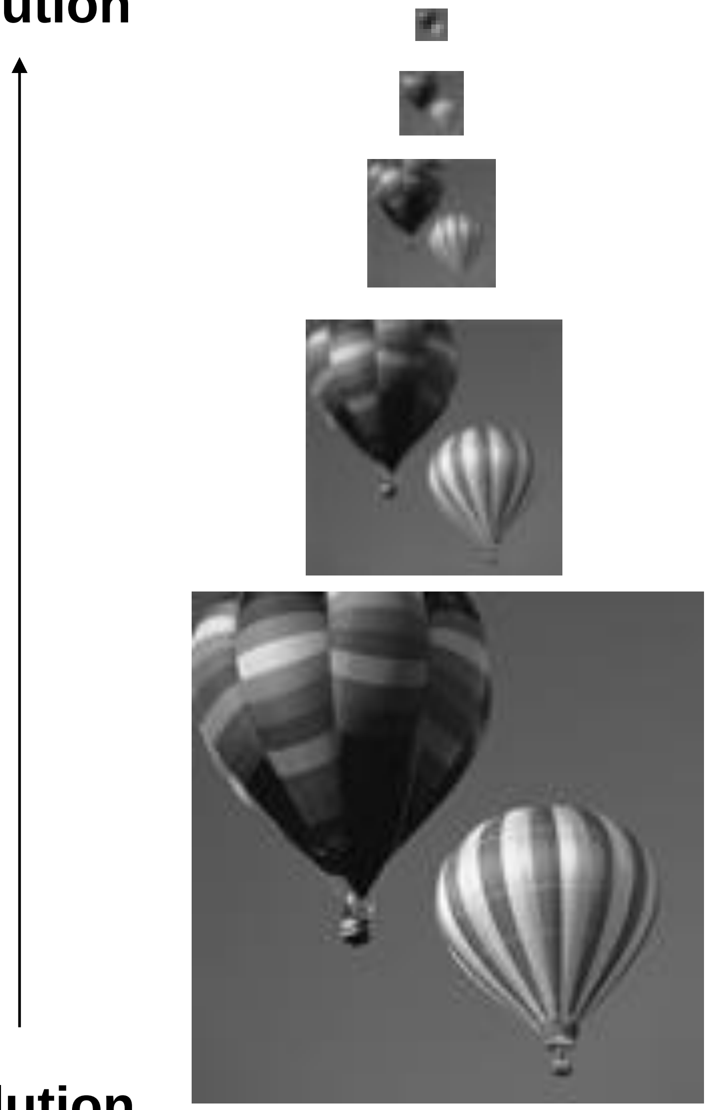




Image Source: Irani & Basri

# Image Pyramid

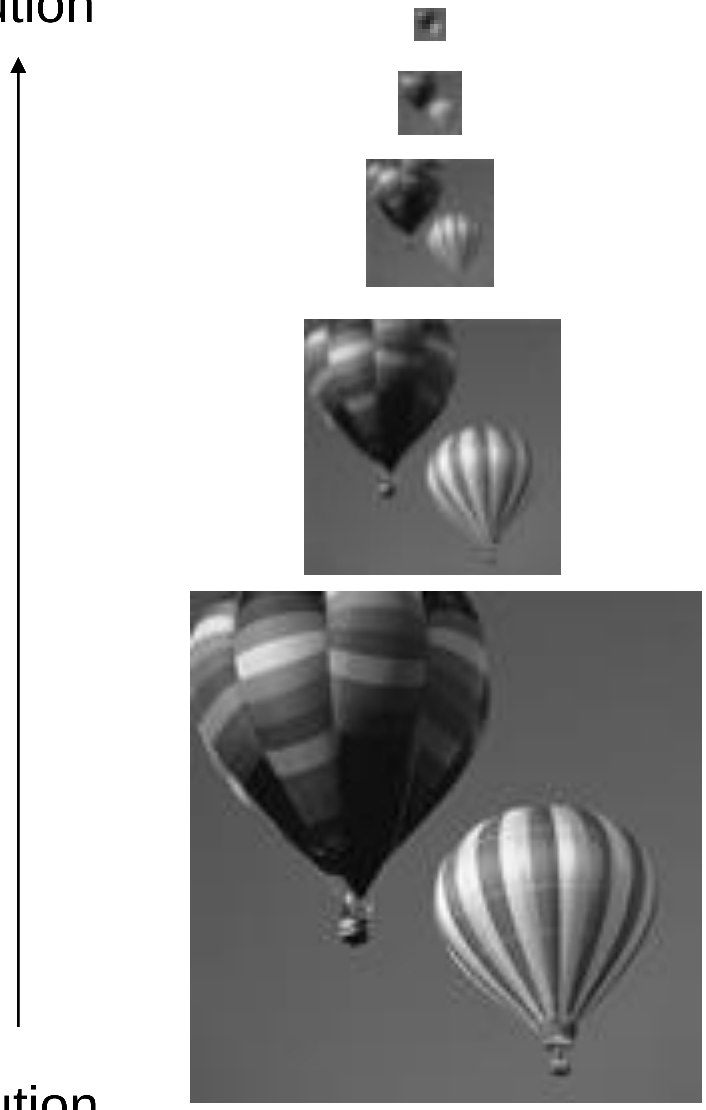
**Low resolution**



**High resolution**

# Image Pyramid

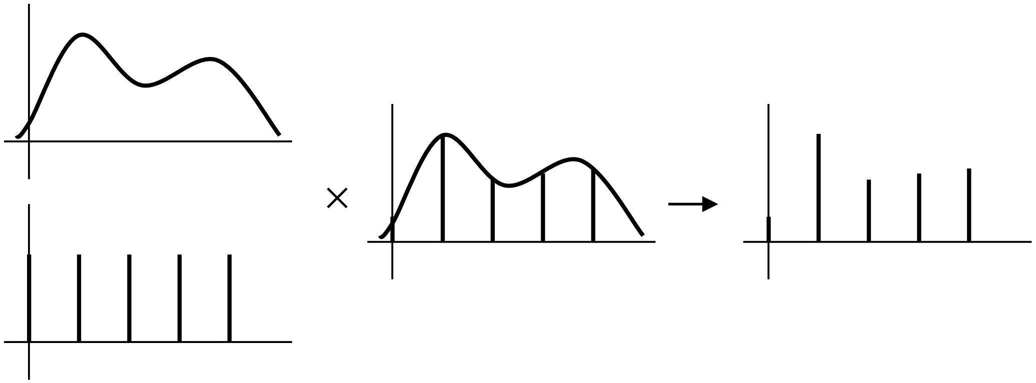
Low resolution



High resolution

# Fourier Interpretation: Discrete Sampling

- Sampling in the spatial domain is like multiplying with a spike function.



- Sampling in the frequency domain is like...

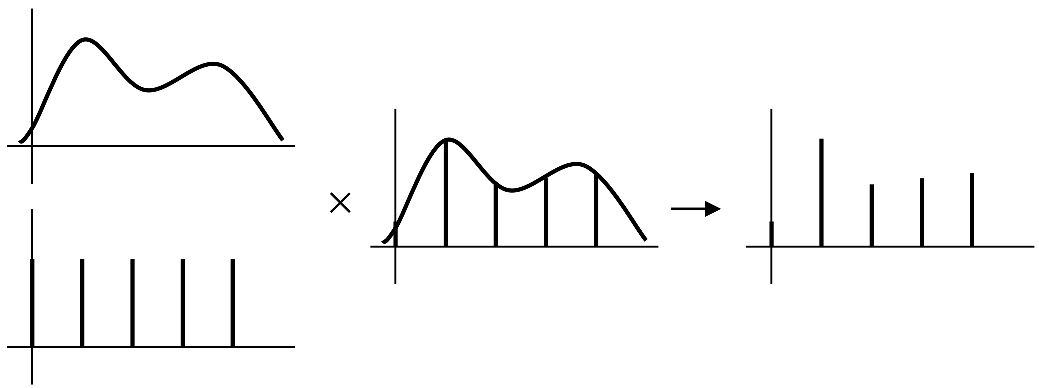
**?**



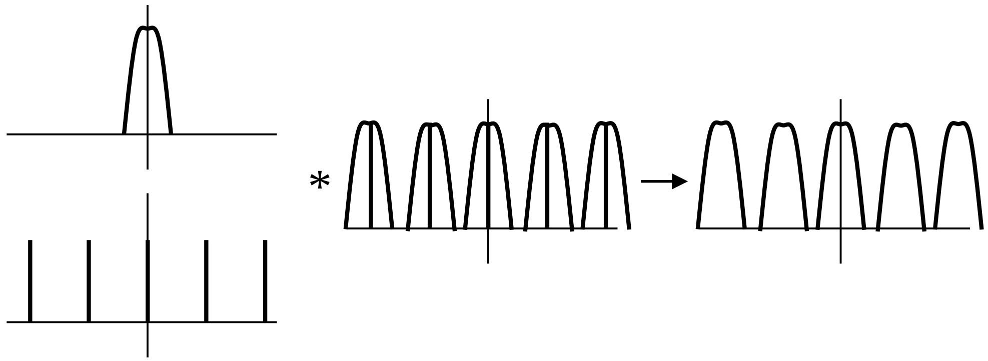
Source: S. Chenney

# Fourier Interpretation: Discrete Sampling

- Sampling in the spatial domain is like multiplying with a spike function.



- Sampling in the frequency domain is like convolving with a spike function.




Source: S. Chenney

# Sampling and Aliasing

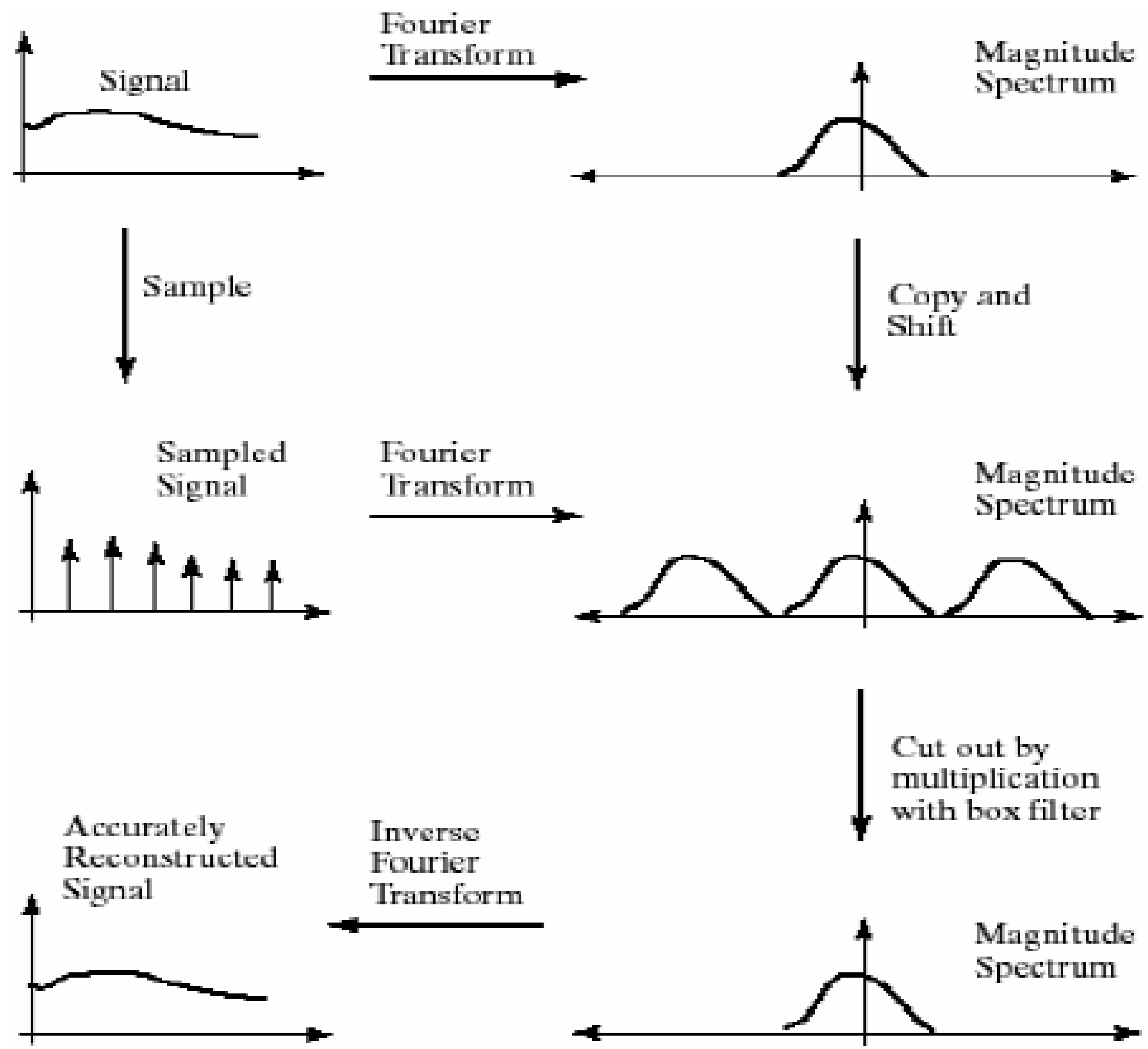
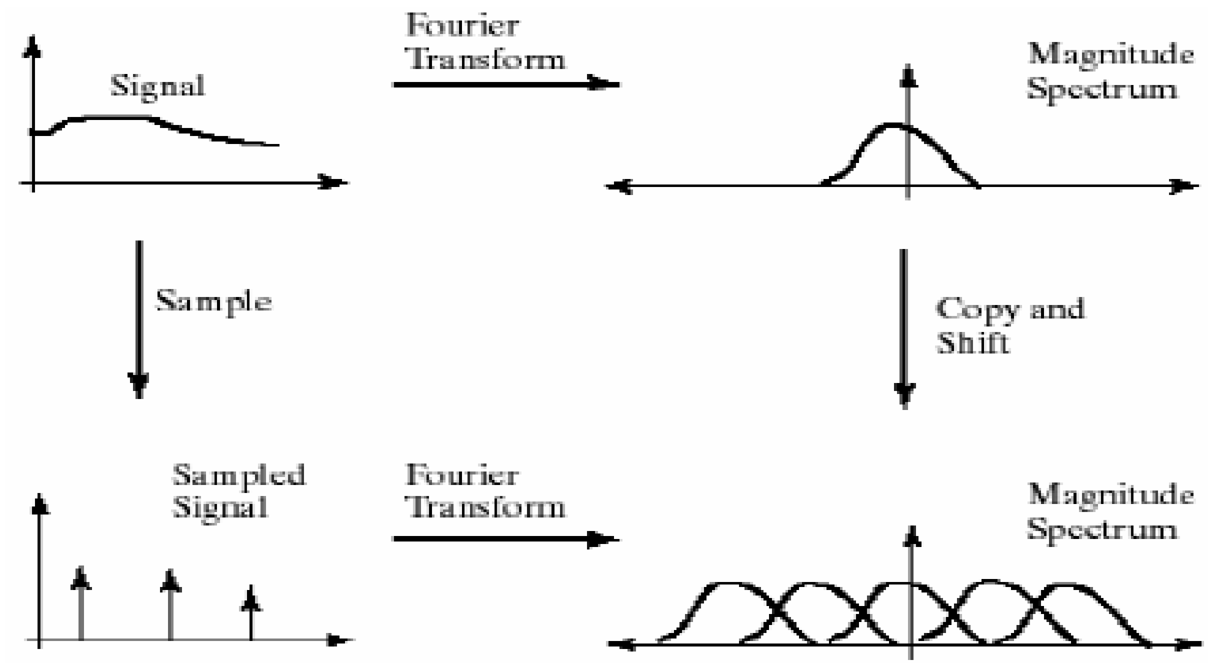




Image Source: Forsyth & Ponce

# Sampling and Aliasing




- Nyquist theorem:
  - In order to recover a certain frequency $f$, we need to sample with at least $2f$.
  - This corresponds to the point at which the transformed frequency spectra start to overlap (the Nyquist limit)



Image Source: Forsyth & Ponce

# Sampling and Aliasing

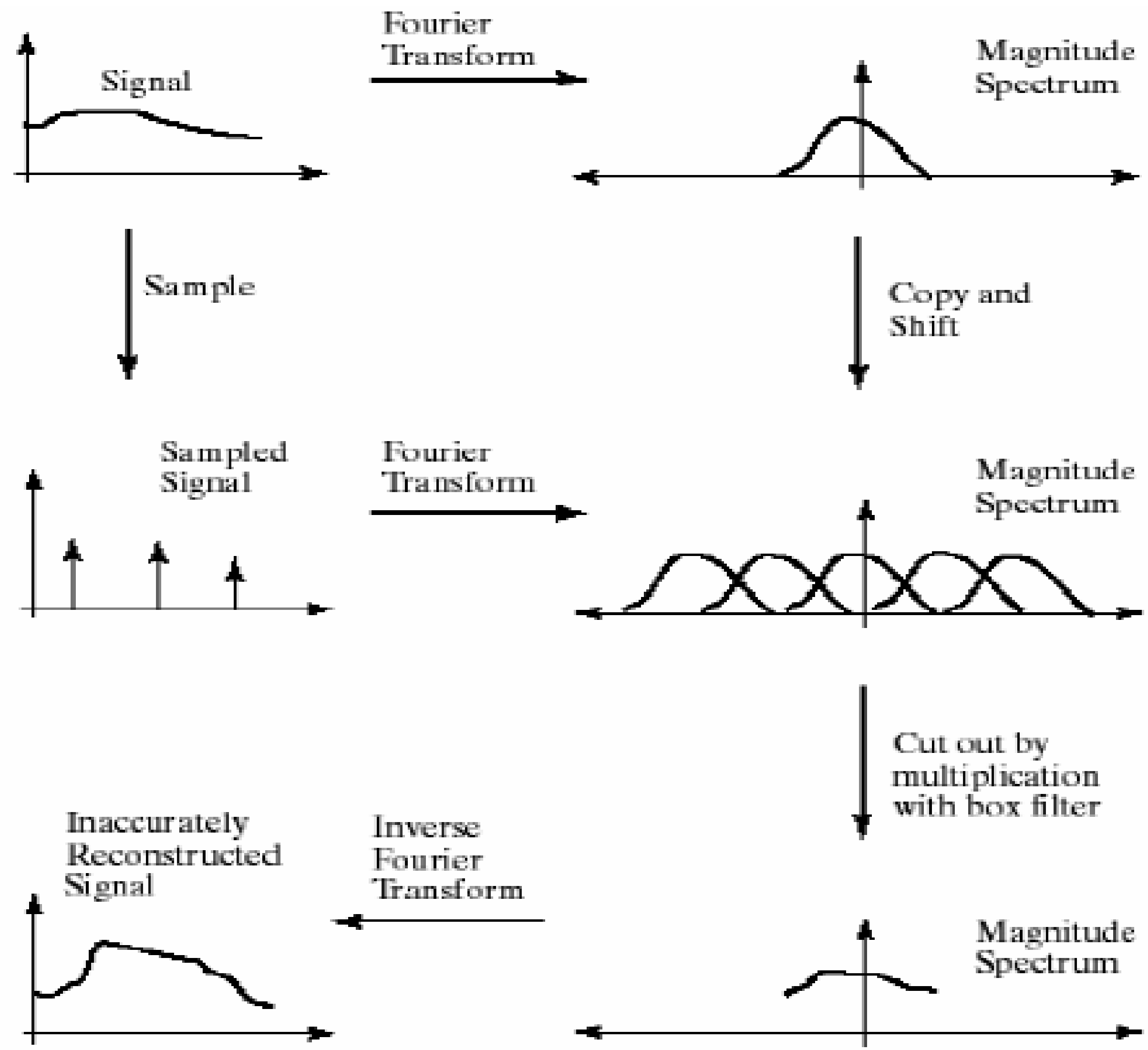




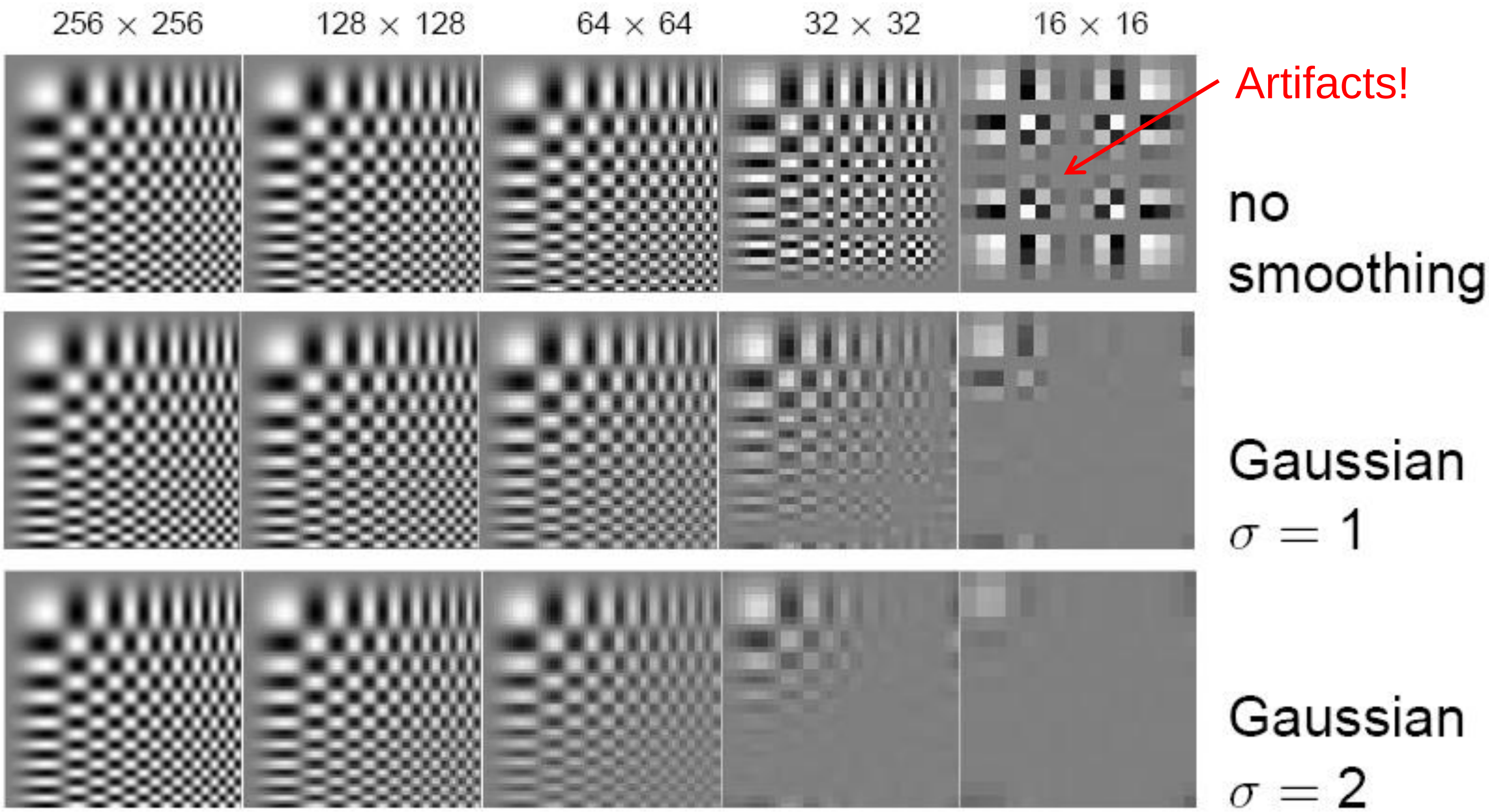

Image Source: Forsyth & Ponce

# Aliasing in Graphics




Image Source: Alexej Efros

# Resampling with Prior Smoothing



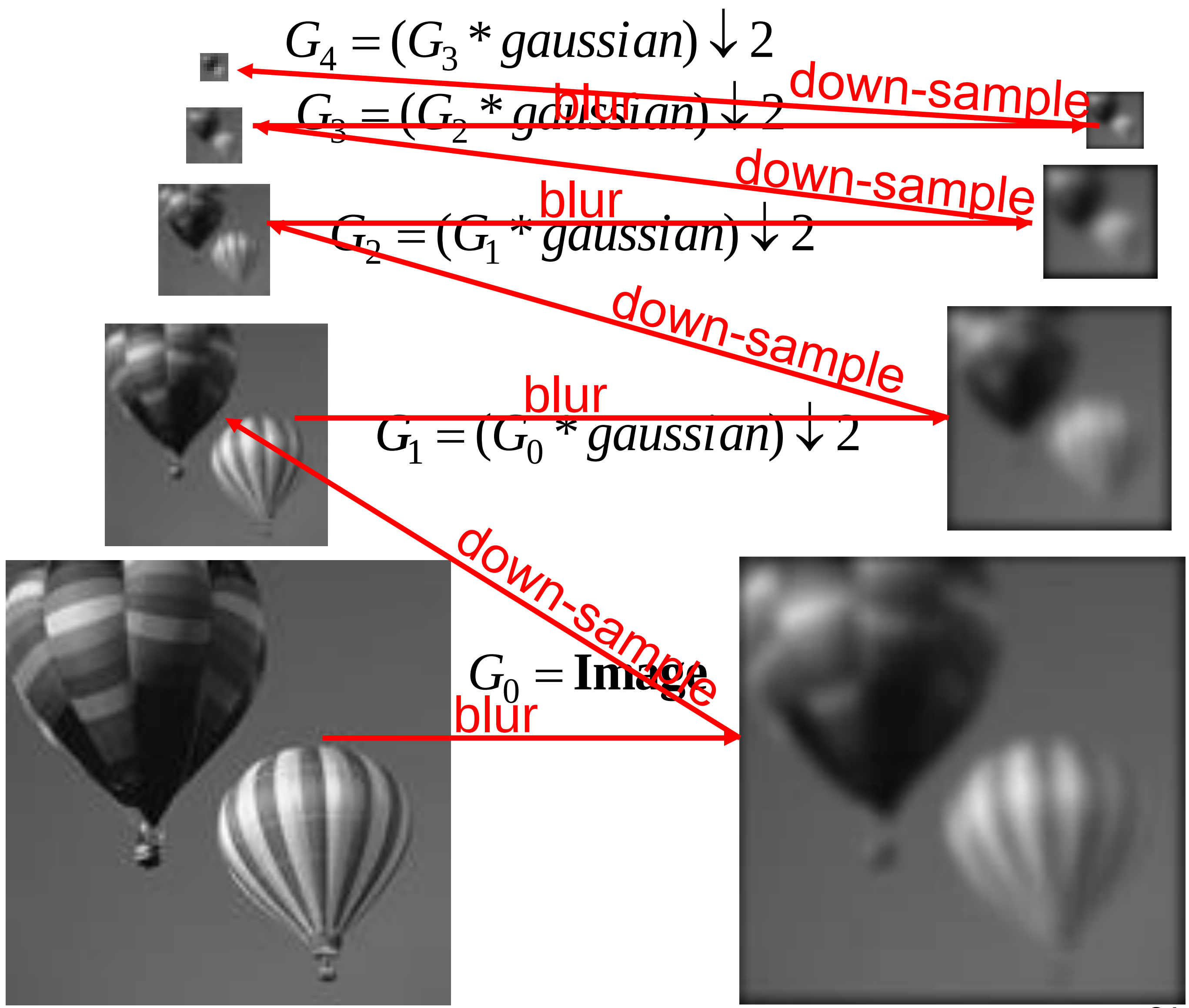

- Note: We cannot recover the high frequencies, but we can avoid artifacts by smoothing before resampling.


Image Source: Forsyth & Ponce

# The Gaussian Pyramid

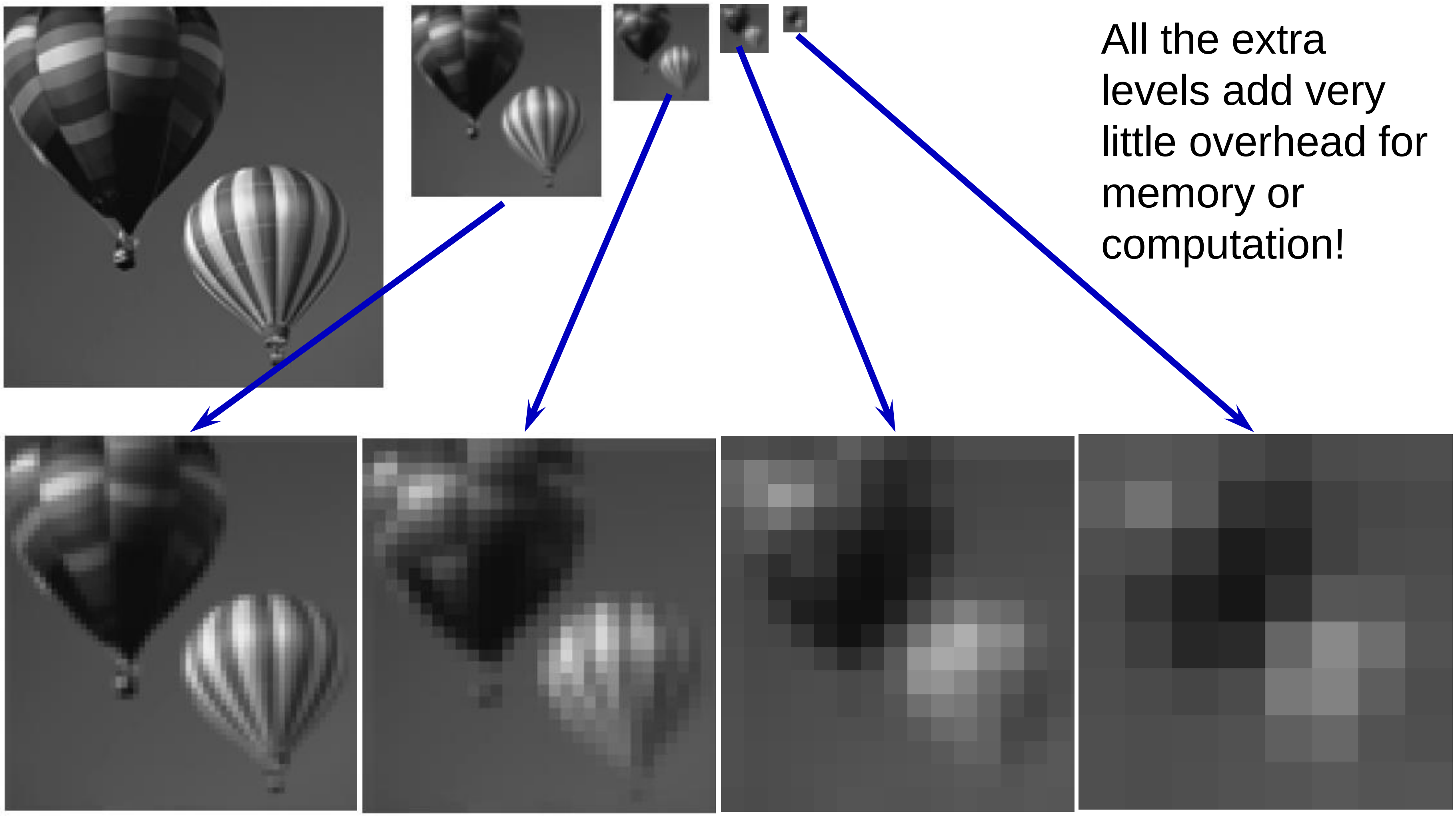
Low resolution

High resolution




Source: Irani & Basri

# Gaussian Pyramid – Stored Information



All the extra levels add very little overhead for memory or computation!



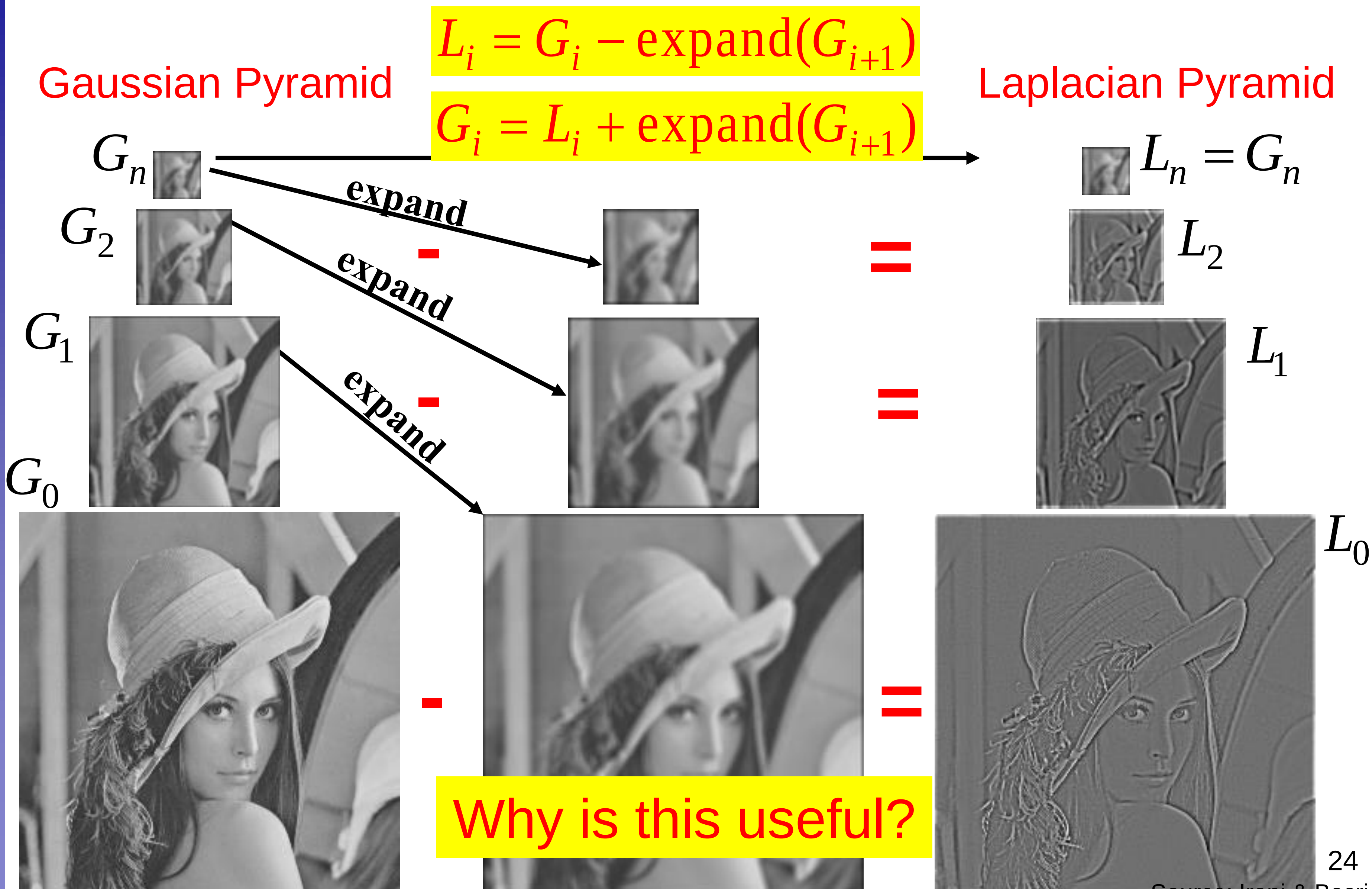
Source: Irani & Basri

# Summary: Gaussian Pyramid

- Construction: create each level from previous one
  - Smooth and sample
- Smooth with Gaussians, in part because
  - a Gaussian*Gaussian = another Gaussian
  - $G(\sigma_1) * G(\sigma_2) = G(\mathrm{sqrt}(\sigma_1^{2} + \sigma_2^{2}))$
- Gaussians are low-pass filters, so the representation is redundant once smoothing has been performed.

  $\Rightarrow$ There is no need to store smoothed images at the full original resolution.

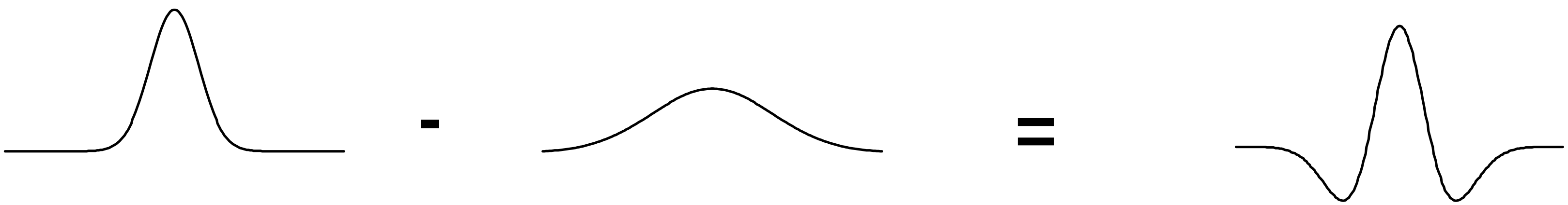
# The Laplacian Pyramid

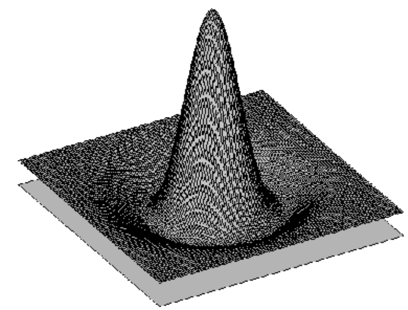





Source: Irani & Basri

# Laplacian ~ Difference of Gaussian





DoG = Difference of Gaussians

Cheap approximation – no derivatives needed.

# Topics of This Lecture

- Recap: Linear Filters
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- Filters as templates
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  - Derivatives of Gaussian
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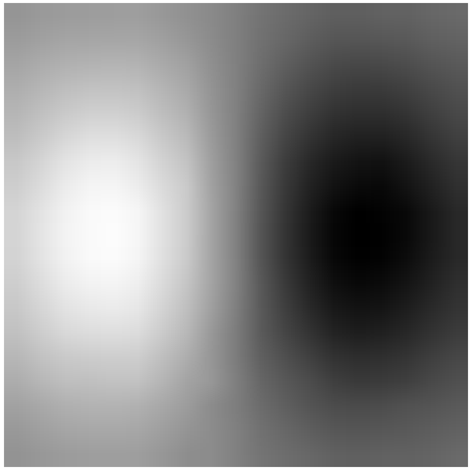
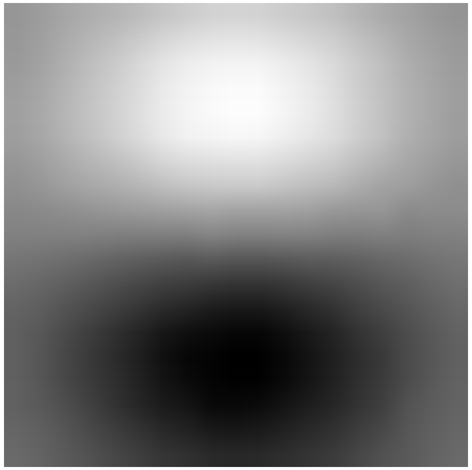
# Note: Filters are Templates

- Applying a filter at some point can be seen as taking a dot-product between the image and some vector.
- Filtering the image is a set of dot products.

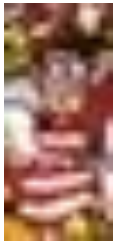
- Insight
  - Filters look like the effects they are intended to find.
  - Filters find effects they look like.





# Where's Waldo?

Scene



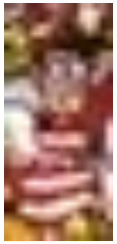
Template



Slide credit: Kristen Grauman

# Where’s Waldo?

Detected template



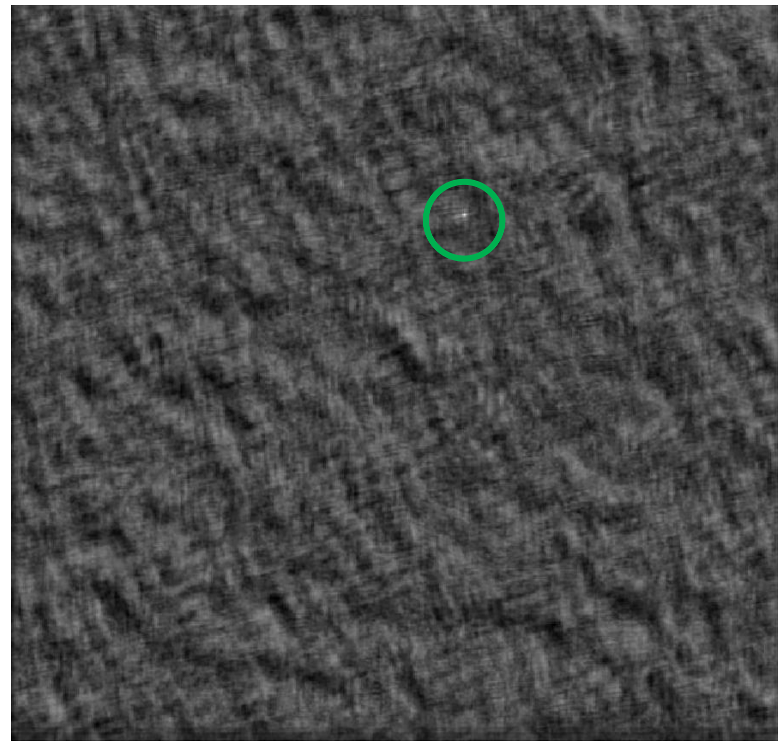
Template



Slide credit: Kristen Grauman

# Where's Waldo?

Detected template



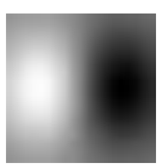
Correlation map

# Correlation as Template Matching

- Think of filters as a dot product of the filter vector with the image region
  - Now measure the angle between the vectors

$$a \cdot b = |a||b|\cos\theta \qquad \cos\theta = \frac{a \cdot b}{|a||b|}$$

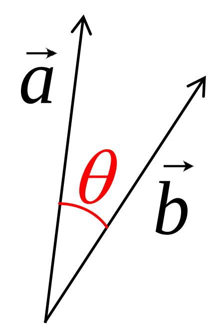
  - Angle (similarity) between vectors can be measured by normalizing the length of each vector to 1 and taking the dot product.



Template

·

Image region




Vector interpretation

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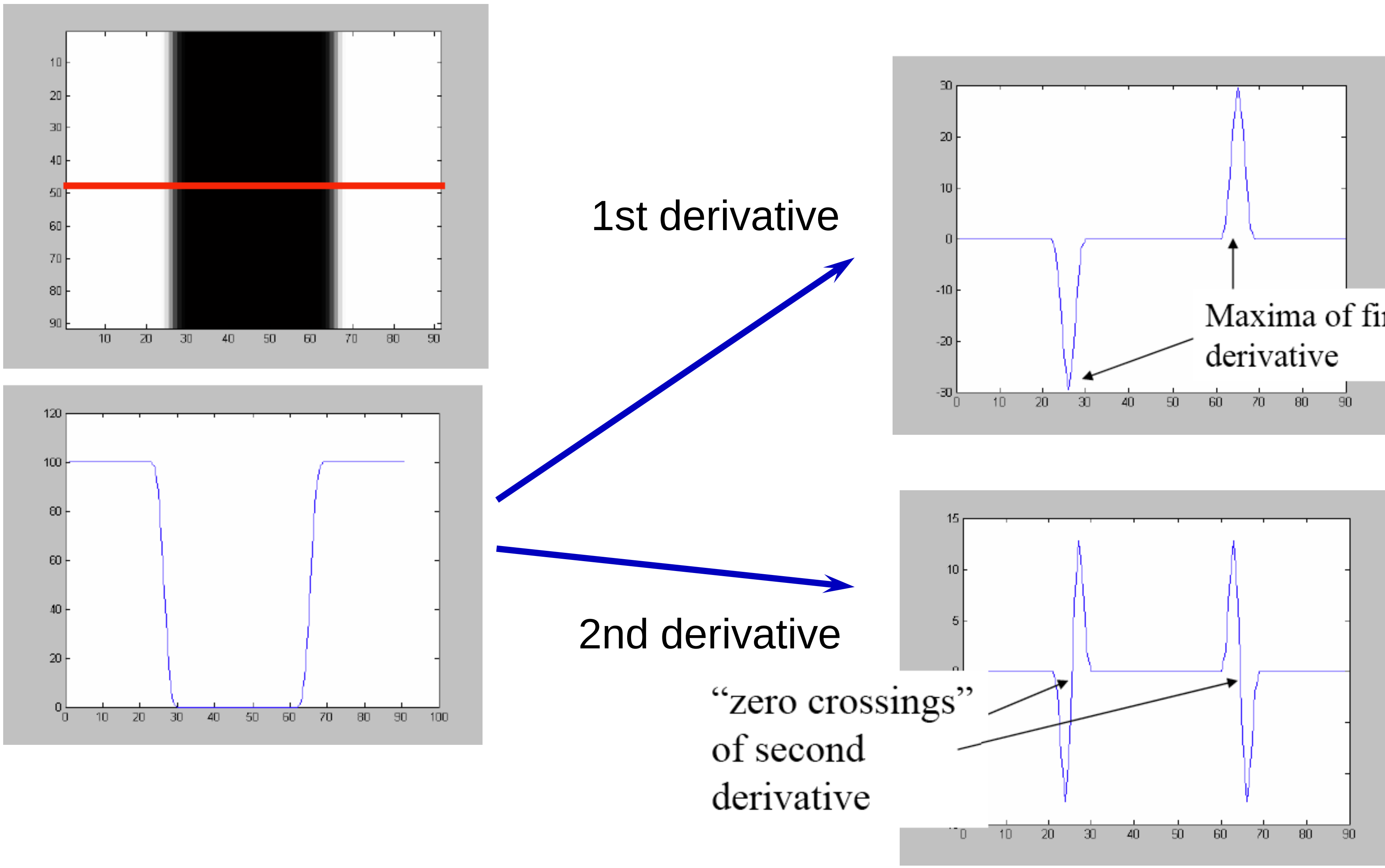
# Derivatives and Edges…



# Differentiation and Convolution

- For the 2D function $f(x,y)$, the partial derivative is:

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\varepsilon \to 0} \frac{f(x+\varepsilon, y) - f(x,y)}{\varepsilon}$$

- For discrete data, we can approximate this using finite differences:

$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f(x+1, y) - f(x,y)}{1}$$

- To implement the above as convolution, what would be the associated filter?

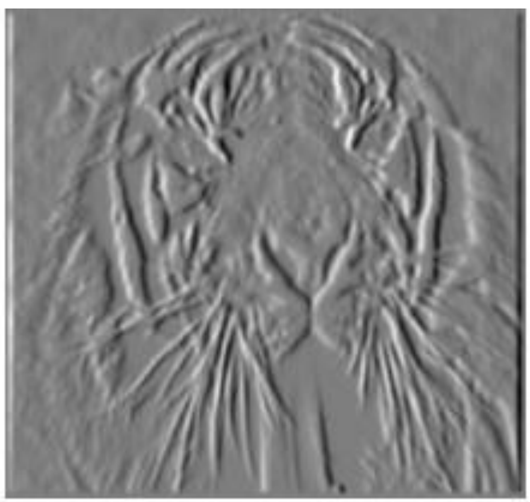
| | |
|---|---|
| **1** | **-1** |



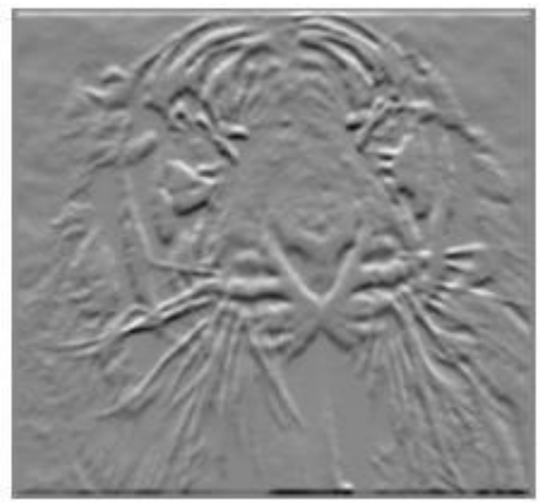
Slide credit: Kristen Grauman

# Partial Derivatives of an Image

$$\frac{\partial f(x,y)}{\partial x}$$



| | |
|---|---|
| **-1** | **1** |



$$\frac{\partial f(x,y)}{\partial y}$$

| |
|---|
| **-1** |
| **1** |

**?**

**or**

| |
|---|
| **1** |
| **-1** |

Which one shows changes with respect to x?

# Assorted Finite Difference Filters

**Prewitt:** $M_x =$

| | | |
|---|---|---|
| -1 | 0 | 1 |
| -1 | 0 | 1 |
| -1 | 0 | 1 |

; $M_y =$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 0 | 0 |
| -1 | -1 | -1 |

**Sobel:** $M_x =$

| | | |
|---|---|---|
| -1 | 0 | 1 |
| -2 | 0 | 2 |
| -1 | 0 | 1 |

; $M_y =$

| | | |
|---|---|---|
| 1 | 2 | 1 |
| 0 | 0 | 0 |
| -1 | -2 | -1 |

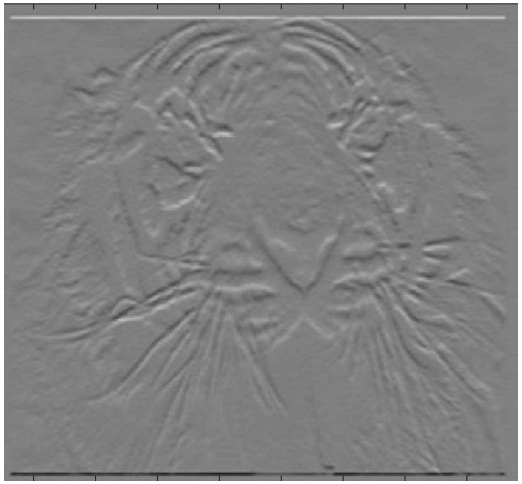
**Roberts:** $M_x =$

| | |
|---|---|
| 0 | 1 |
| -1 | 0 |

; $M_y =$

| | |
|---|---|
| 1 | 0 |
| 0 | -1 |

```
>> My = fspecial('sobel');
>> outim = imfilter(double(im), My);
>> imagesc(outim);
>> colormap gray;
```

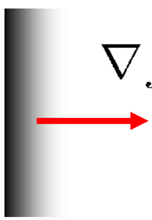


# Image Gradient

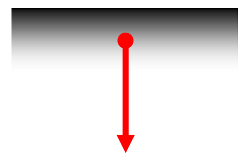
- The gradient of an image:

$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

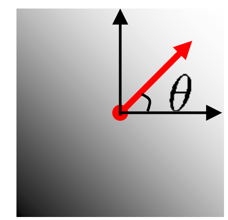
- The gradient points in the direction of most rapid intensity change



$$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$$



$$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$$




$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

- The gradient direction (orientation of edge normal) is given by:

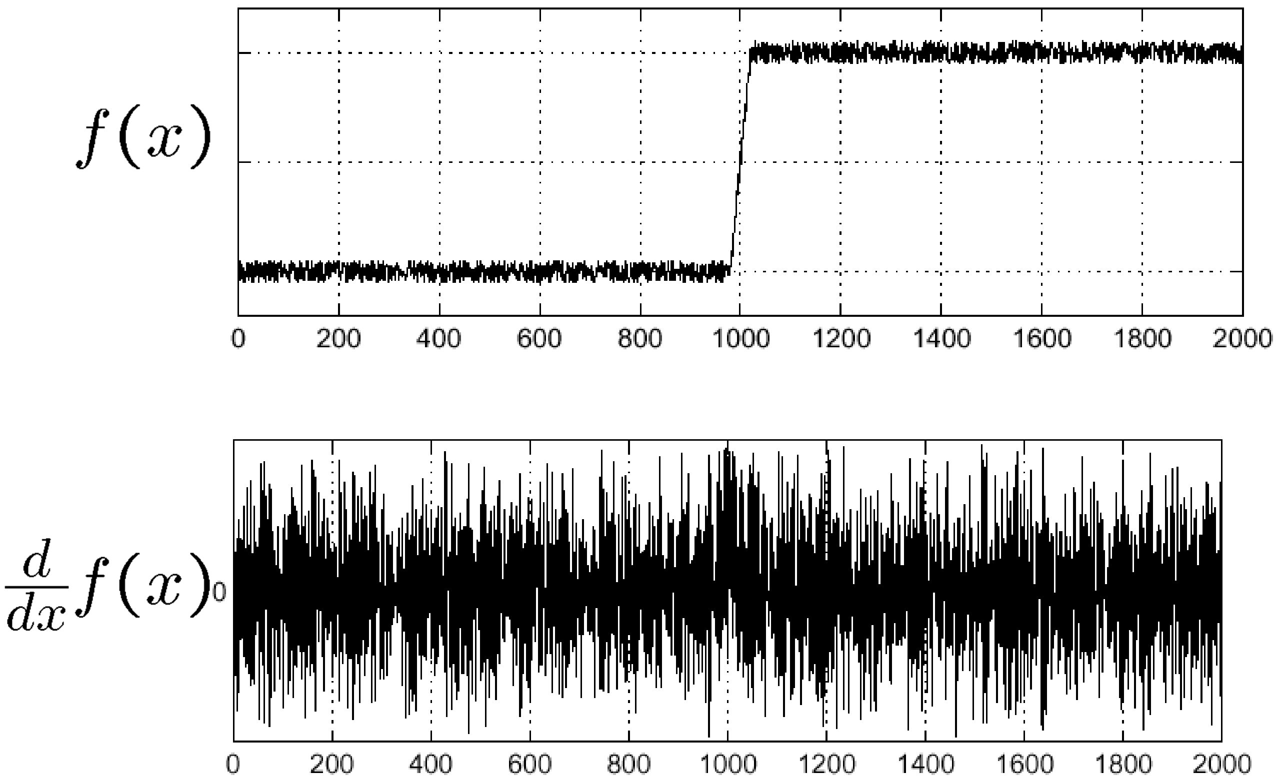
$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

- The edge strength is given by the gradient magnitude

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

# Effect of Noise

- Consider a single row or column of the image
  - Plotting intensity as a function of position gives a signal




Where is the edge?

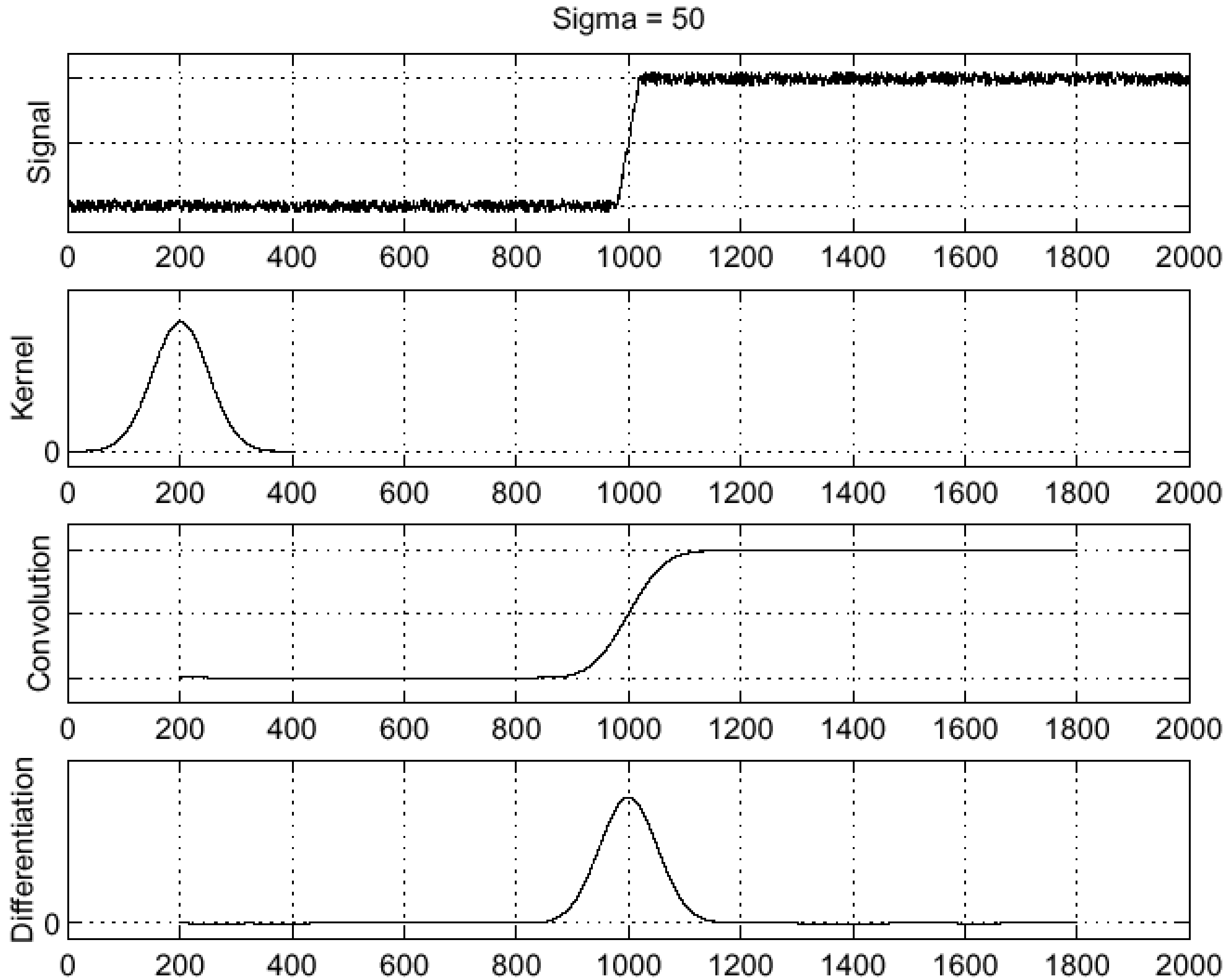
# Solution: Smooth First

$f$

$h$

$h \star f$

$\frac{\partial}{\partial x}(h \star f)$




Where is the edge? Look for peaks in $\frac{\partial}{\partial x}(h \star f)$

# Derivative Theorem of Convolution

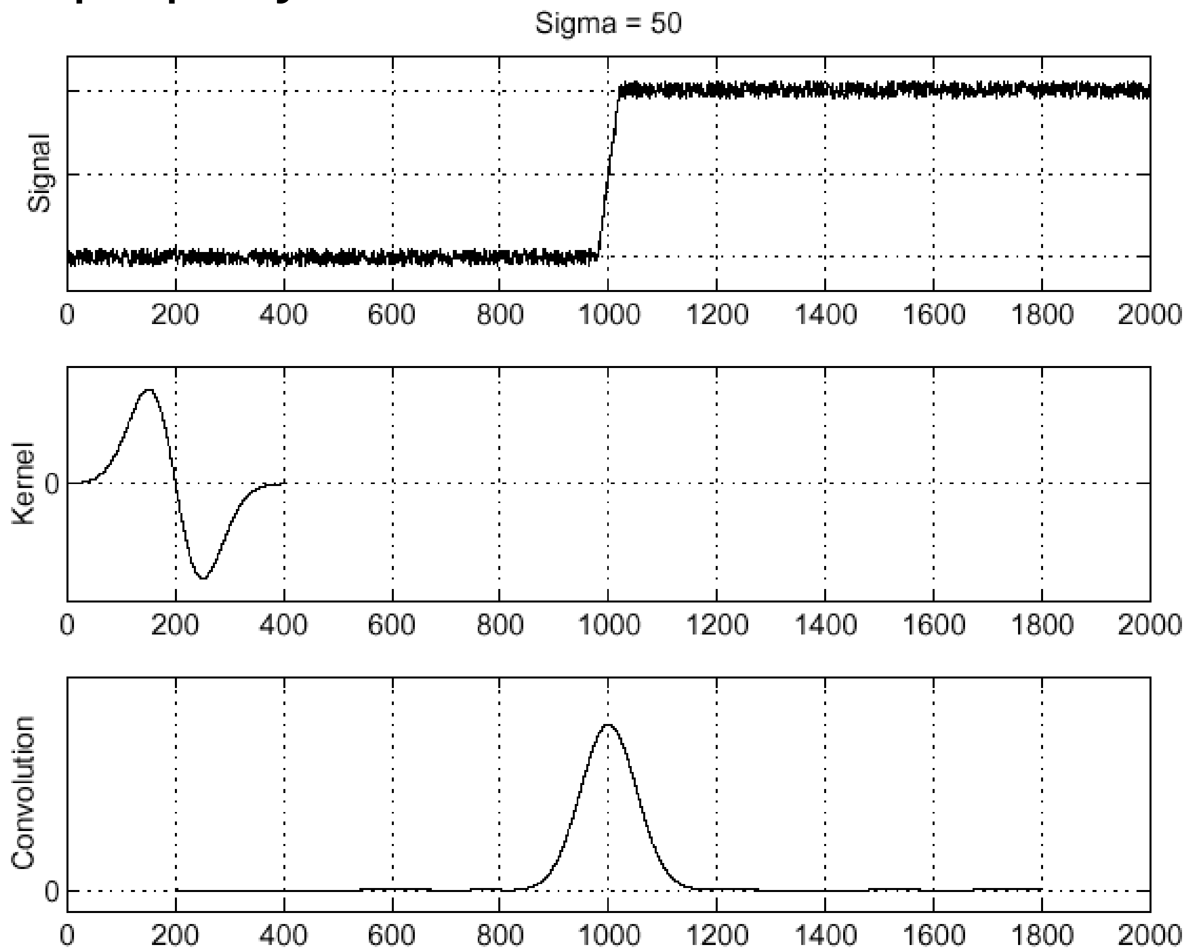
$$\frac{\partial}{\partial x}(h \star f) = (\frac{\partial}{\partial x}h) \star f$$

- Differentiation property of convolution.

$f$

$\frac{\partial}{\partial x}h$

$(\frac{\partial}{\partial x}h) \star f$

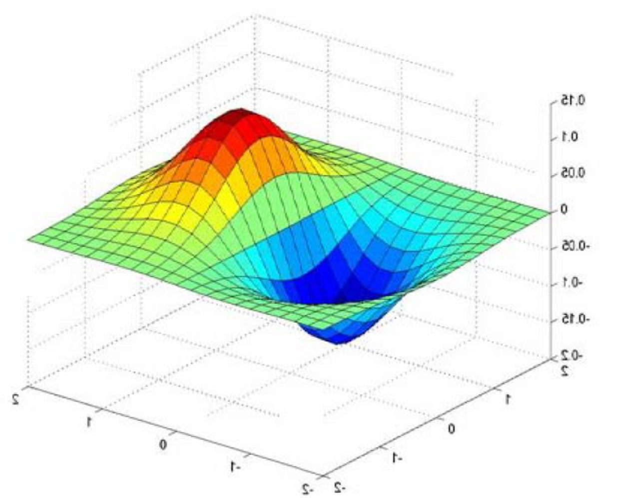


# Derivative of Gaussian Filter

$$g * (h * I) = (g * h) * I$$

$$\begin{bmatrix} 1 & -1 \end{bmatrix} * \begin{bmatrix} 0.0030 & 0.0133 & 0.0219 & 0.0133 & 0.0030 \\ 0.0133 & 0.0596 & 0.0983 & 0.0596 & 0.0133 \\ 0.0219 & 0.0983 & 0.1621 & 0.0983 & 0.0219 \\ 0.0133 & 0.0596 & 0.0983 & 0.0596 & 0.0133 \\ 0.0030 & 0.0133 & 0.0219 & 0.0133 & 0.0030 \end{bmatrix}$$

Why is this preferable?



Slide adapted from Kristen Grauman

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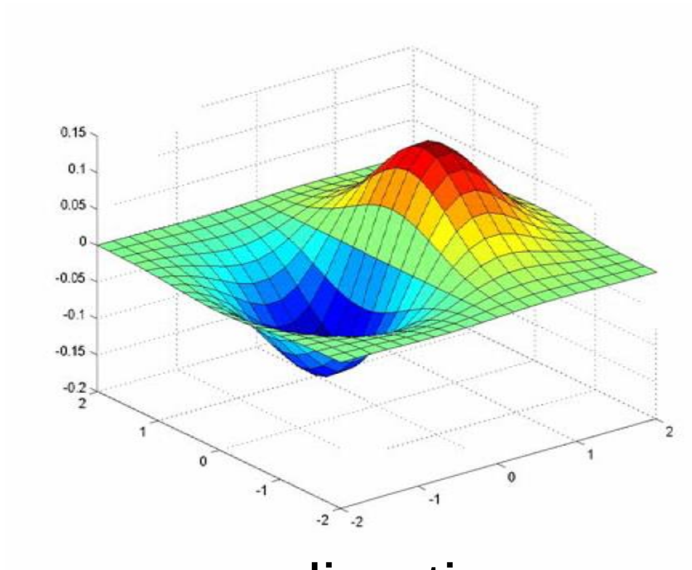
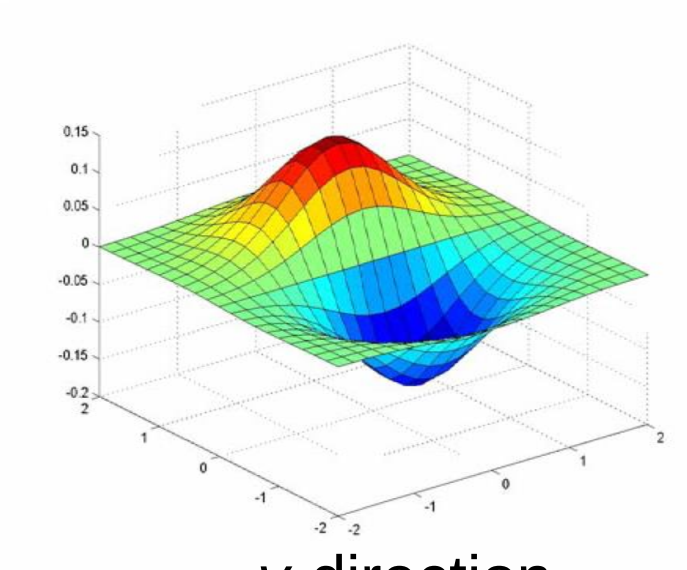
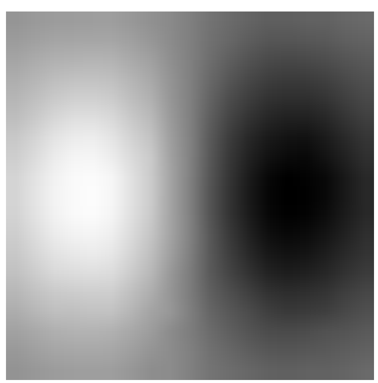
# Derivative of Gaussian Filters

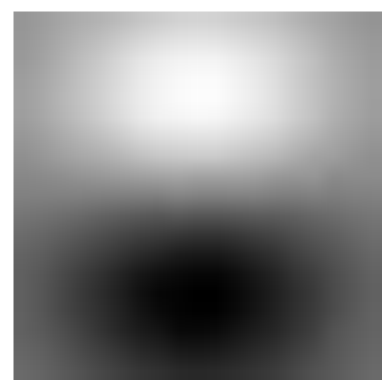



*x*-direction




*y*-direction





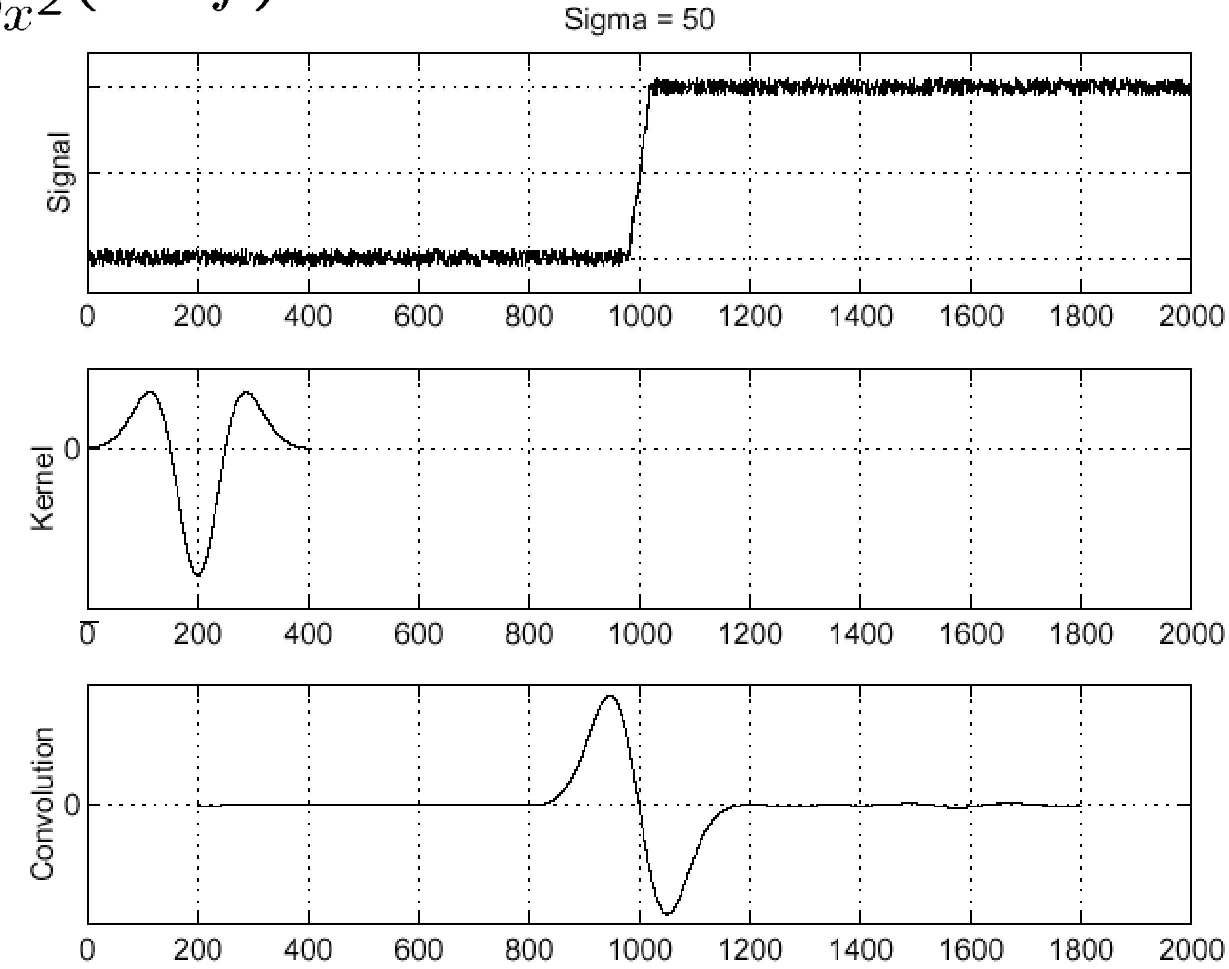
# Laplacian of Gaussian (LoG)

- Consider $\frac{\partial^2}{\partial x^2}(h \star f)$

$f$

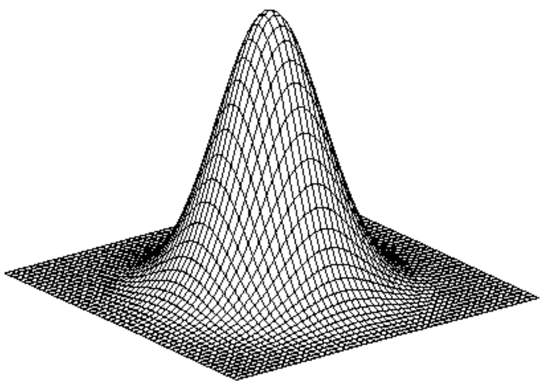
$\frac{\partial^2}{\partial x^2}h$

$(\frac{\partial^2}{\partial x^2}h) \star f$



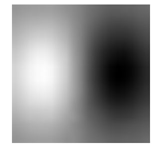

Where is the edge? Zero-crossings of bottom graph

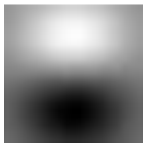
# Summary: 2D Edge Detection Filters

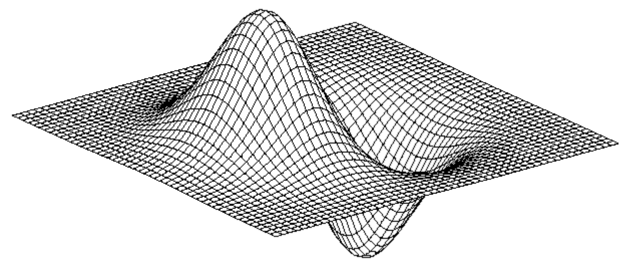


Gaussian

$$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



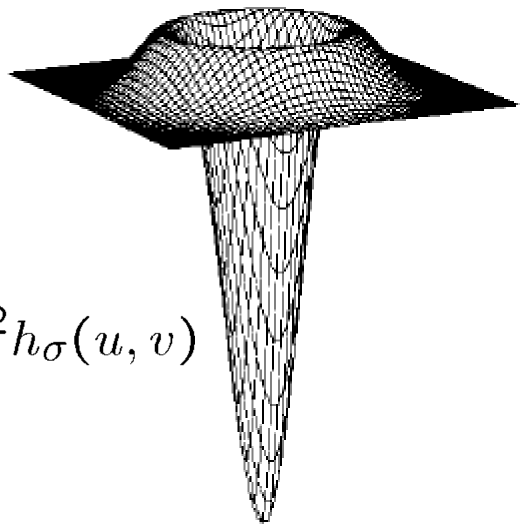




Derivative of Gaussian

$$\frac{\partial}{\partial x} h_\sigma(u,v)$$

Laplacian of Gaussian



$$\nabla^2 h_\sigma(u,v)$$

- $\nabla^2$ is the Laplacian operator:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

Slide credit: Kristen Grauman

# Topics of This Lecture

- Recap: Linear Filters
- Multi-Scale representations
  - How to properly rescale an image?
- Filters as templates
  - Correlation as template matching
- Image gradients
  - Derivatives of Gaussian
- Edge detection
  - Canny edge detector

# Edge Detection

- Goal: map image from 2D array of pixels to a set of curves or line segments or contours.
- Why?

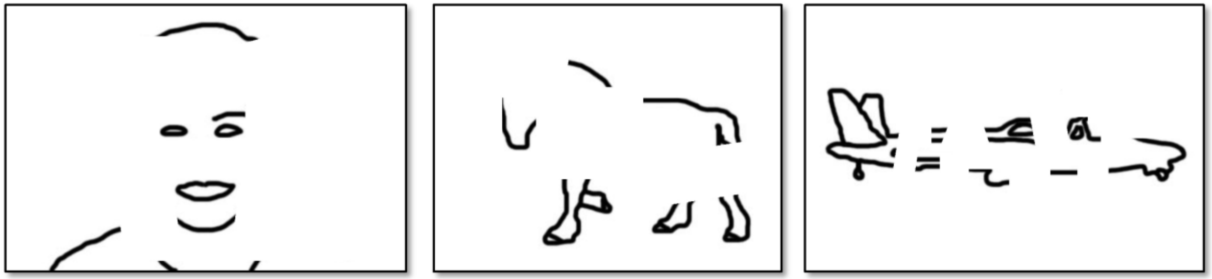


Figure from J. Shotton et al., PAMI 2007

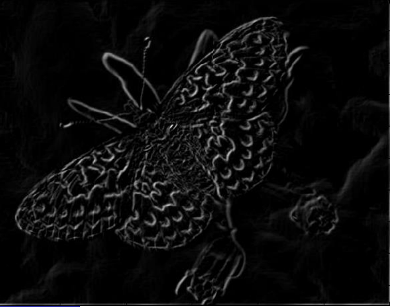
- Main idea: look for strong gradients, post-process

# Designing an Edge Detector

- Criteria for an "optimal" edge detector:
  - Good detection: the optimal detector should minimize the probability of false positives (detecting spurious edges caused by noise), as well as that of false negatives (missing real edges).
  - Good localization: the edges detected should be as close as possible to the true edges.
  - Single response: the detector should return one point only for each true edge point; that is, minimize the number of local maxima around the true edge.



Source: Li Fei-Fei



# Gradients → Edges

Primary edge detection steps

1. Smoothing: suppress noise
2. Edge enhancement: filter for contrast
3. Edge localization
   - Determine which local maxima from filter output are actually edges vs. noise
   - Thresholding, thinning

- Two issues
  - At what scale do we want to extract structures?
  - How sensitive should the edge extractor be?

# Scale: Effect of σ on Derivatives

$\sigma$ = 1 pixel $\sigma$ = 3 pixels

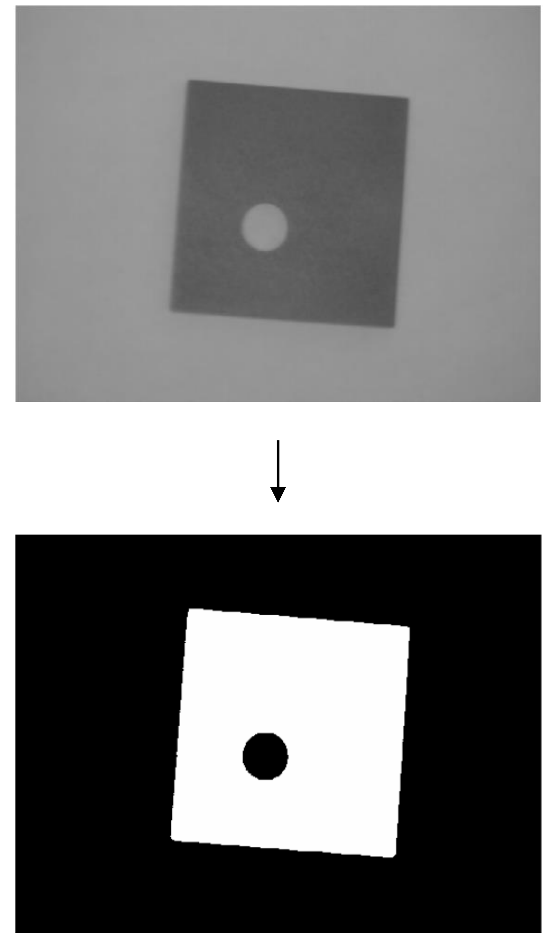
- The apparent structures differ depending on Gaussian's scale parameter.

⇒ Larger values: larger-scale edges detected
⇒ Smaller values: finer features detected

# Sensitivity: Compare to Thresholding

- Choose a threshold $t$
- Set any pixels less than $t$ to zero (off).
- Set any pixels greater than or equal $t$ to one (on).

$$F_T[i,j] = \begin{cases} 1, & \text{if } F[i,j] \geq t \\ 0, & \text{otherwise} \end{cases}$$

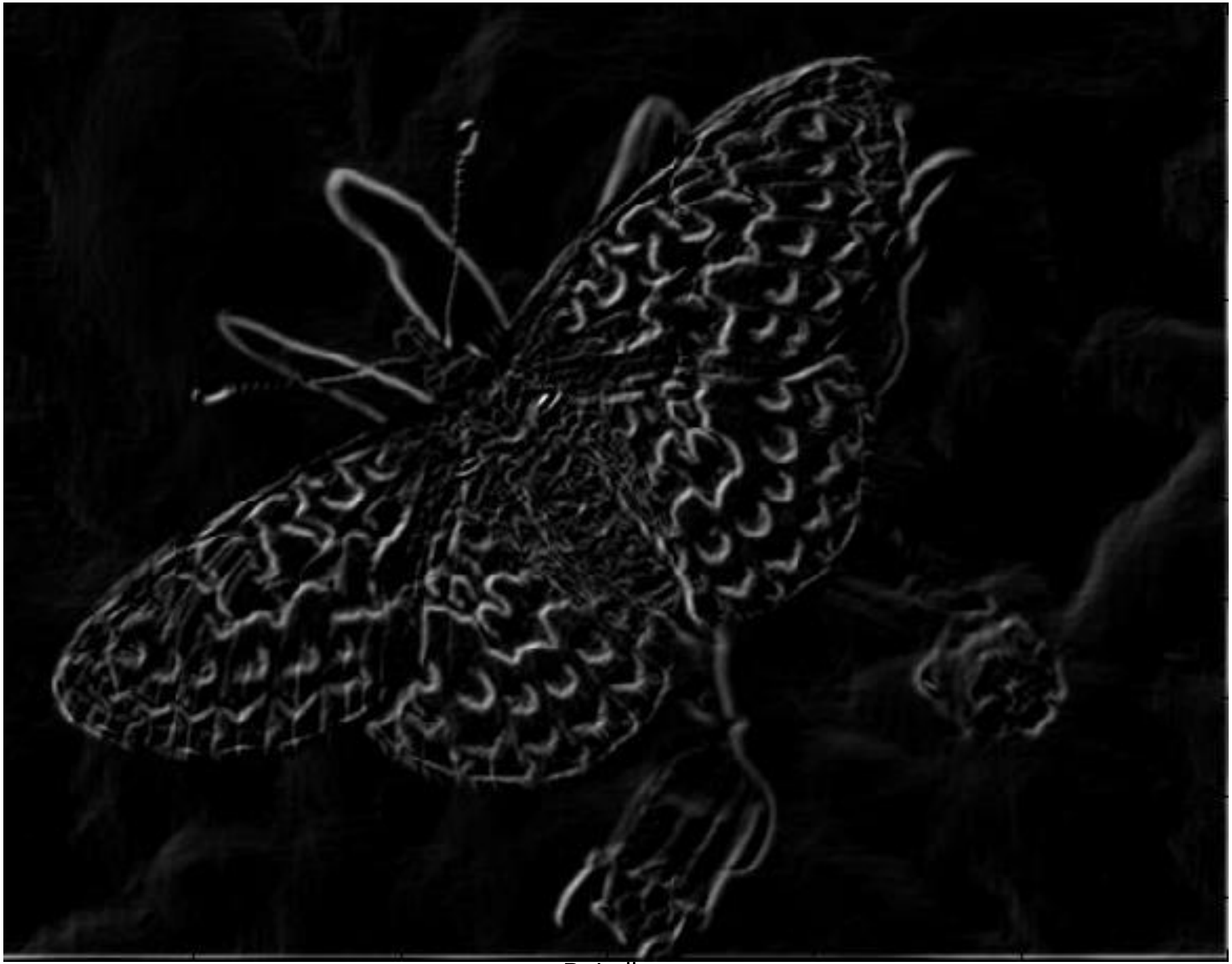


# Original Image



Slide credit: Kristen Grauman

# Gradient Magnitude Image



Slide credit: Kristen Grauman

# Thresholding with a Lower Threshold

Slide credit: Kristen Grauman

# Thresholding with a Higher Threshold

# Canny Edge Detector

- A very widely used edge detector in computer vision
- Theoretical model: step-edges corrupted by additive Gaussian noise
- Canny has shown that the first derivative of the Gaussian closely approximates the operator that optimizes the product of *signal-to-noise ratio* and localization.

J. Canny, A Computational Approach To Edge Detection, *IEEE Trans. Pattern Analysis and Machine Intelligence*, 8:679-714, 1986.



**Source: Li Fei-Fei**

# The Canny Edge Detector

Original image



Slide credit: Kristen Grauman

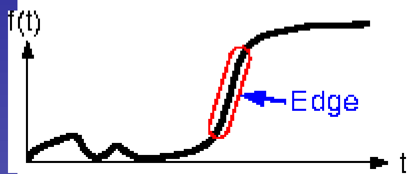
# The Canny Edge Detector

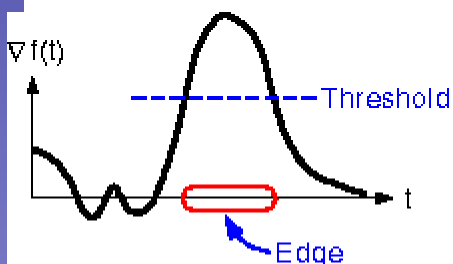
Gradient magnitude

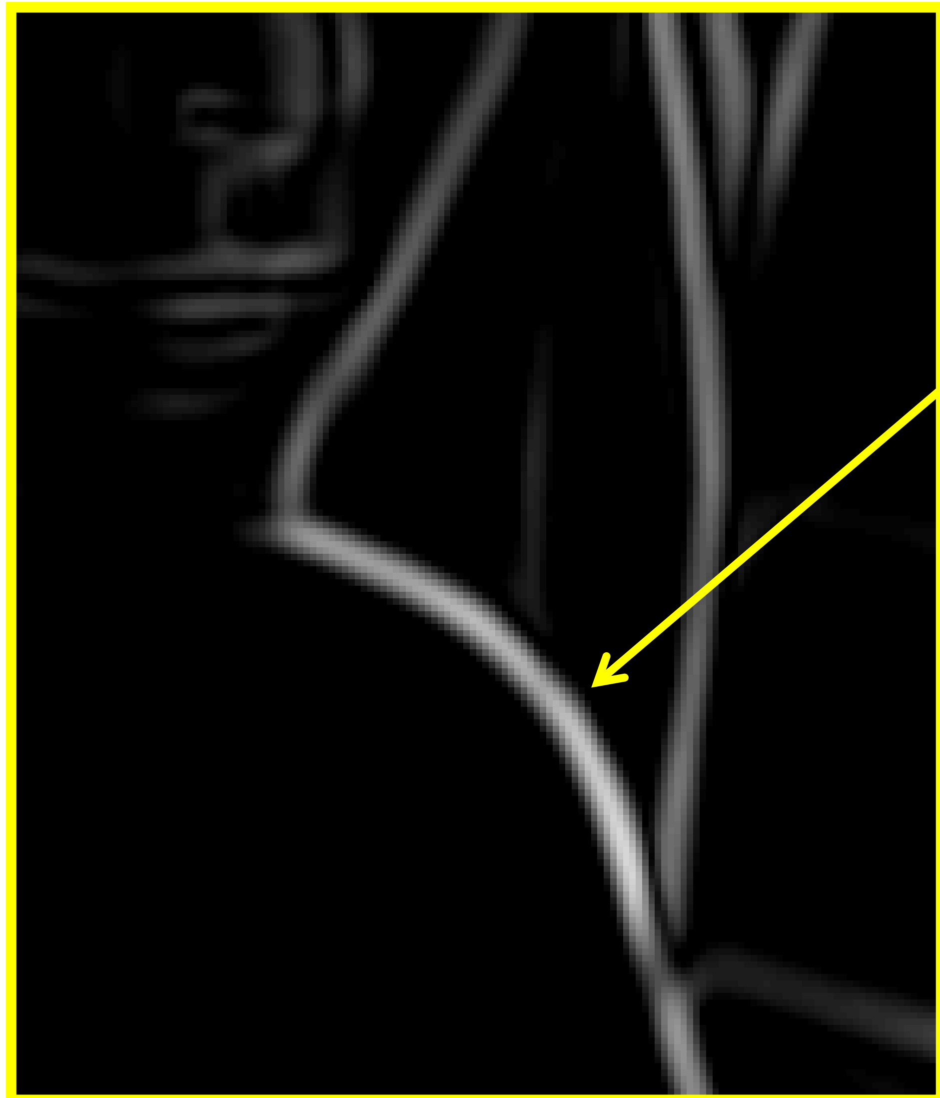


Slide credit: Kristen Grauman

# The Canny Edge Detector

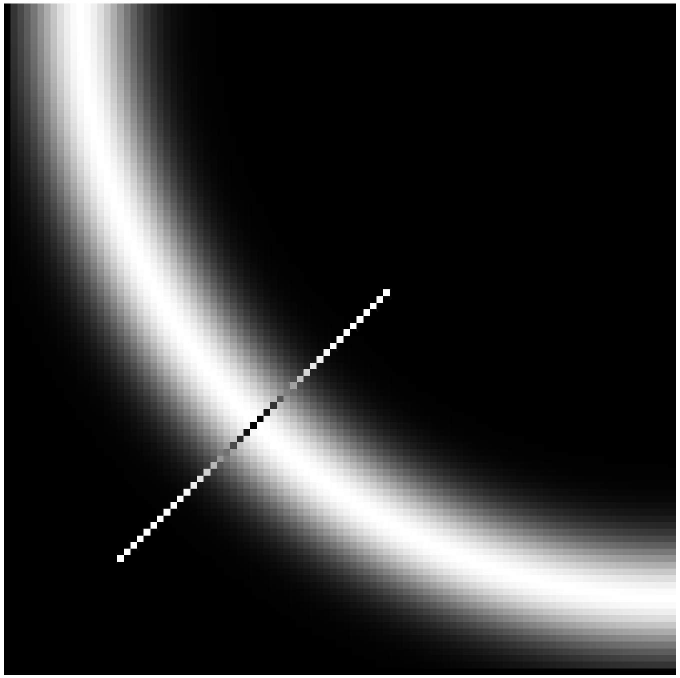


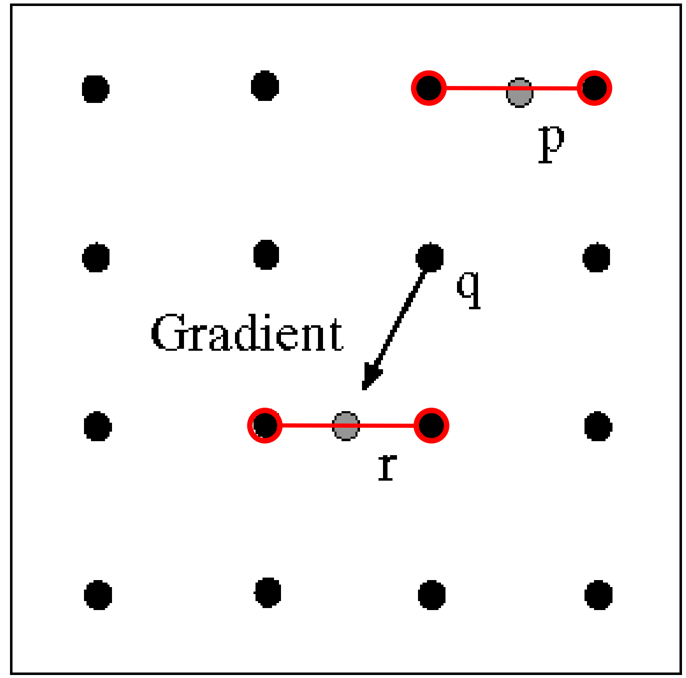



How to turn these thick regions of the gradient into single-pixel curves?

# Non-Maximum Suppression






- Check if pixel is local maximum along gradient direction, select single max across width of the edge
  - I.e., keep $q$ iff $\mathrm{Mag}(q) > \mathrm{Mag}(p)$ and $\mathrm{Mag}(q) > \mathrm{Mag}(r)$.
  - Requires checking interpolated pixels p and r

$\Rightarrow$ Linear interpolation based on gradient direction



Source: Forsyth & Ponce

# The Canny Edge Detector

Problem: pixels along this edge didn't survive the thresholding.

Thinning
(non-maximum suppression)

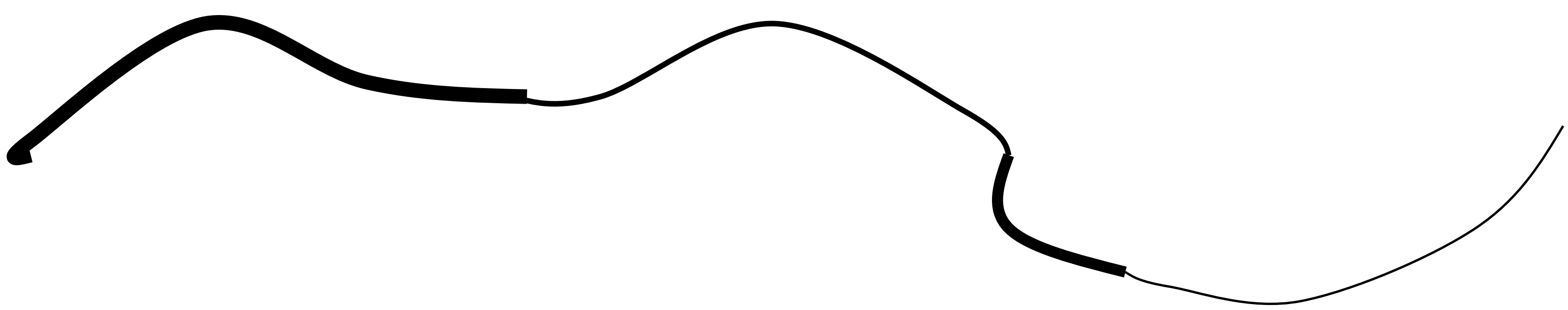


Slide credit: Kristen Grauman

# Solution: Hysteresis Thresholding

- Hysteresis: A lag or momentum factor
- Idea: Maintain two thresholds $k_{high}$ and $k_{low}$
  - Use $k_{high}$ to find strong edges to start edge chain
  - Use $k_{low}$ to find weak edges which continue edge chain
- Typical ratio of thresholds is roughly

$$k_{high} \; / \; k_{low} = 2$$




Source: D. Lowe, S. Seitz

# Hysteresis Thresholding

Original image

High threshold
(strong edges)

Low threshold
(weak edges)

courtesy of G. Loy

Hysteresis threshold





**Source: L. Fei-Fei**

# Summary: Canny Edge Detector

1. Filter image with derivative of Gaussian
2. Find magnitude and orientation of gradient
3. Non-maximum suppression:
   - Thin multi-pixel wide "ridges" down to single pixel width
4. Linking and thresholding (hysteresis):
   - Define two thresholds: low and high
   - Use the high threshold to start edge curves and the low threshold to continue them

- MATLAB:

```
>> edge(image, 'canny');
>> help edge
```

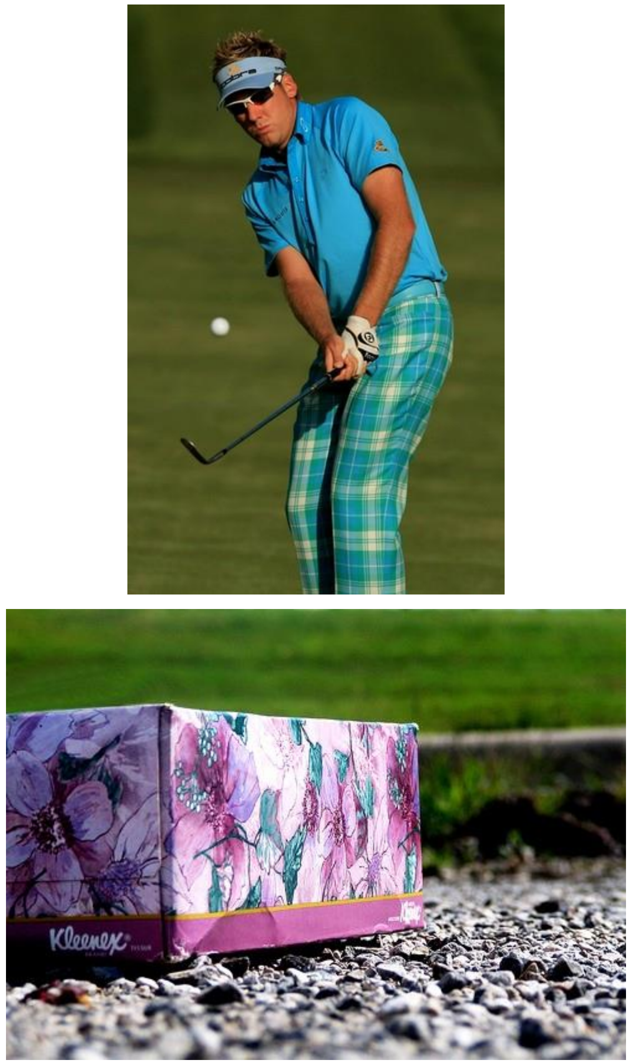




Source: D. Lowe, L. Fei-Fei

# Object Boundaries vs. Edges

Background



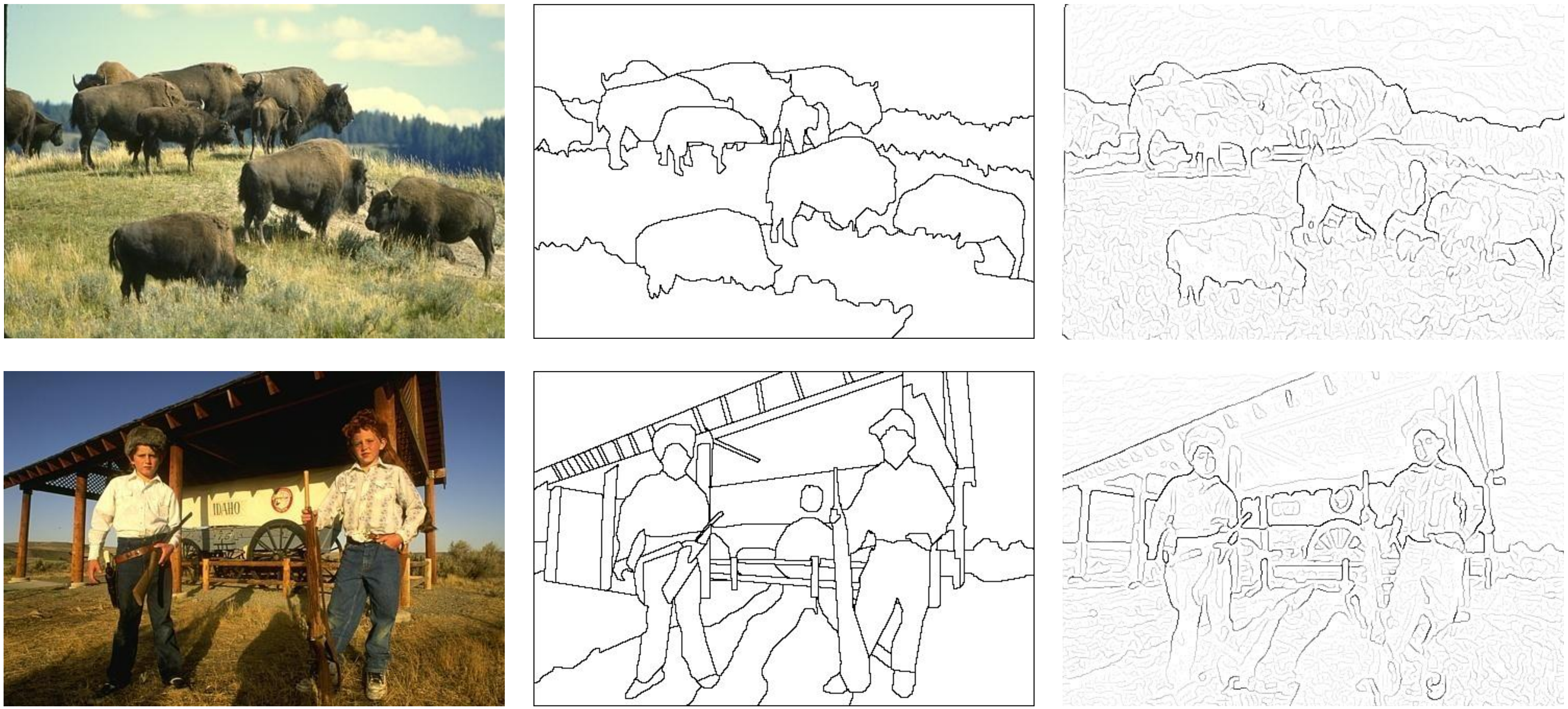

Texture

Shadows

Slide credit: Kristen Grauman

# Edge Detection is Just the Beginning…

Image | Human segmentation | Gradient magnitude



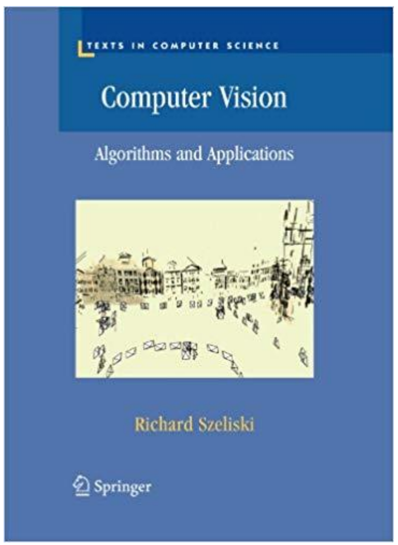
- Berkeley segmentation database:
  http://www.eecs.berkeley.edu/Research/Projects/CS/vision/grouping/segbench/



Source: L. Lazebnik

# References and Further Reading

- Background information on linear filters and edge detection can be found in Chapter 3 of the Szeliski book or in Chapters 7 and 8 of Forsyth & Ponce.




R. Szeliski
Computer Vision – Algorithms and Applications
Springer, 2010

D. Forsyth, J. Ponce,
*Computer Vision – A Modern Approach*.
Prentice Hall, 2003