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# Computer Vision - Lecture 2

## Binary Image Analysis

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## Announcements

- Course webpage
  - <http://www.vision.rwth-aachen.de/teaching/>
  - Slides will be made available on the webpage
- L2P electronic repository
  - Exercises and supplementary materials will be posted on the L2P
- Please subscribe to the lecture on the Campus system!
  - Important to get email announcements and L2P access!
  - Bachelor students please also subscribe

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2

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## Binary Images

- Just two pixel values
- Foreground and background
- Regions of interest

1	1	0	1	1	1	0	1
1	1	0	1	0	1	0	1
1	1	1	1	0	0	0	1
0	0	0	0	0	0	0	1
1	1	1	1	0	1	0	1
0	0	0	1	0	1	0	1
1	1	0	1	0	0	0	1
1	1	0	1	0	1	1	1



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3

Slide credit: Kristen Grauman

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## Uses: Industrial Inspection

Fig. 1 Schematic diagram of marking inspection setup at Texas Instruments

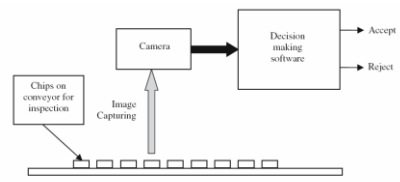


Fig. 2 Inspected image

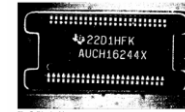
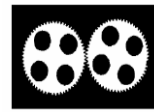


Fig. 3 Raw scan for separating a line



R. Nagarajan et al. "A real time marking inspection scheme for semiconductor industries", 2006

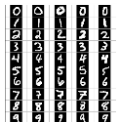
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4


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## Uses: Document Analysis, Text Recognition


Handwritten digits



Natural text (after detection)



Scanned documents



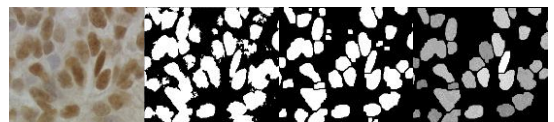
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Source: Till Quack, Martin Rombel

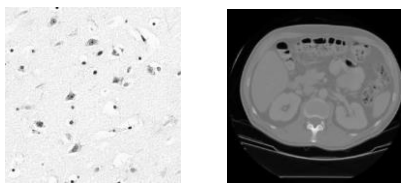
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## Uses: Medical/Bio Data



Source: D. Kim et al., Cytometry 35(1), 1999



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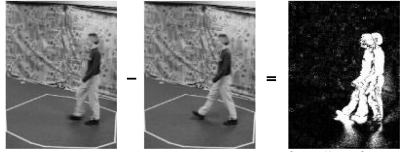
6

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## Uses: Blob Tracking & Motion Analysis

Frame Differencing



Source: Kristen Grauman

Background Subtraction



Source: Tobias Jäggi

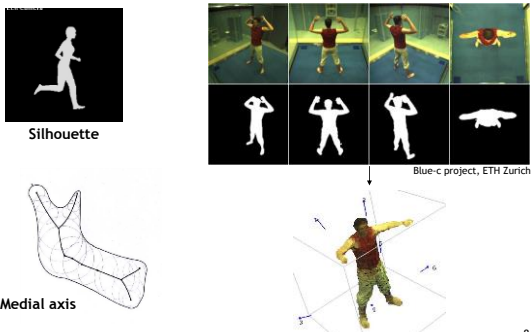
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7

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## Uses: Shape Analysis, Free-Viewpoint Video

Visual Hull Reconstruction



Silhouette

Medial axis

Blue-c project, ETH Zurich

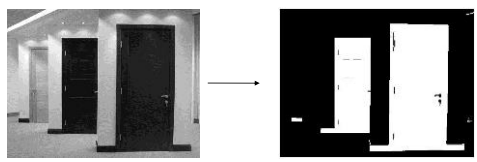
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8

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## Uses: Intensity Based Detection

- Looking for dark pixels...



`fg_pix = find(im < 65);`

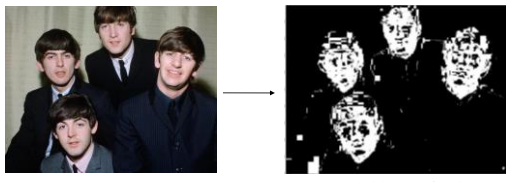
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9

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## Uses: Color Based Detection

- Looking for pixels within a certain color range...



`fg_pix = find(hue > t1 & hue < t2);`

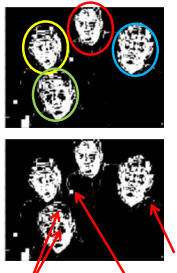
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10

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## Issues

- How to demarcate multiple regions of interest?
  - Count objects
  - Compute further features per object
- What to do with "noisy" binary outputs?
  - Holes
  - Extra small fragments



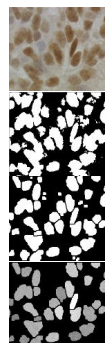
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11

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## Outline of Today's Lecture

- Convert the image into binary form
  - Thresholding
- Clean up the thresholded image
  - Morphological operators
- Extract individual objects
  - Connected Components Labeling
- Describe the objects
  - Region properties

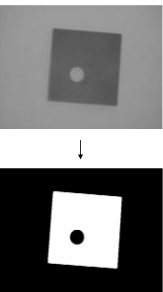


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12

**Thresholding**

- Grayscale image  $\Rightarrow$  Binary mask
- Different variants
  - > One-sided
 
$$F_T[i, j] = \begin{cases} 1, & \text{if } F[i, j] \geq T \\ 0, & \text{otherwise} \end{cases}$$
  - > Two-sided
 
$$F_T[i, j] = \begin{cases} 1, & \text{if } T_1 \leq F[i, j] \leq T_2 \\ 0, & \text{otherwise} \end{cases}$$
  - > Set membership
 
$$F_T[i, j] = \begin{cases} 1, & \text{if } F[i, j] \in Z \\ 0, & \text{otherwise} \end{cases}$$

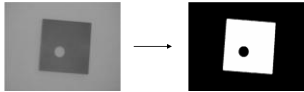


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13

**Selecting Thresholds**

- Typical scenario
  - > Separate an object from a distinct background

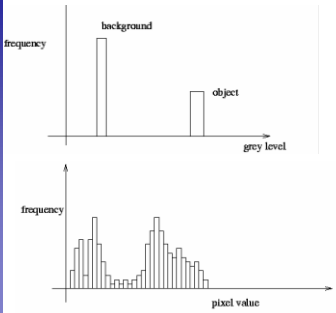


- Try to separate the different grayvalue distributions
  - > Partition a bimodal histogram
  - > Fit a parametric distribution (e.g. Mixture of Gaussians)
  - > Dynamic or local thresholds
- In the following, I will present some simple methods.
  - > We will then see some more general methods in Lecture 6...

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14

**A Nice Case: Bimodal Intensity Histograms**



frequency

background

object

grey level

Ideal histogram, light object on dark background

frequency

Actual observed histogram with noise

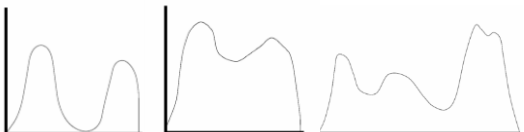
pixel value

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15

**Not so Nice Cases...**

- How to separate those?



Two distinct modes    Overlapping modes    Multiple modes

- Threshold selection is difficult in the general case
  - > Domain knowledge often helps
  - > E.g. Fraction of text on a document page ( $\Rightarrow$  histogram quantile)
  - > E.g. Size of objects/structure elements

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**Global Binarization [Otsu'79]**

- Search for the threshold  $T$  that minimizes the within-class variance  $\sigma_{within}$  of the two classes separated by  $T$ 

$$\sigma_{within}^2(T) = n_1(T)\sigma_1^2 + n_2(T)\sigma_2^2(T)$$
- where
 
$$n_1(T) = |\{I_{(x,y)} < T\}|, \quad n_2(T) = |\{I_{(x,y)} \geq T\}|$$
- This is the same as maximizing the between-class variance  $\sigma_{between}$ 


$$\begin{aligned} \sigma_{between}^2(T) &= \sigma^2 - \sigma_{within}^2(T) \\ &= n_1(T)n_2(T) [\mu_1(T) - \mu_2(T)]^2 \end{aligned}$$

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17

**Algorithm**

1. Precompute a cumulative grayvalue histogram  $h$ .
2. For each potential threshold  $T$ 
  - a) Separate the pixels into two clusters according to  $T$
  - b) Look up  $n_1, n_2$  in  $h$  and compute both cluster means
  - c) Compute  $\sigma_{between}^2(T) = n_1(T)n_2(T) [\mu_1(T) - \mu_2(T)]^2$
3. Choose
 
$$T^* = \arg \max_T [\sigma_{between}^2(T)]$$



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18


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## Local Binarization [Niblack'86]

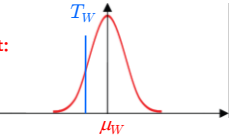
- Estimate a local threshold within a small neighborhood window  $W$

$$T_W = \mu_W + k \cdot \sigma_W$$

where  $k \in [-1, 0]$  is a user-defined parameter.



Effect:

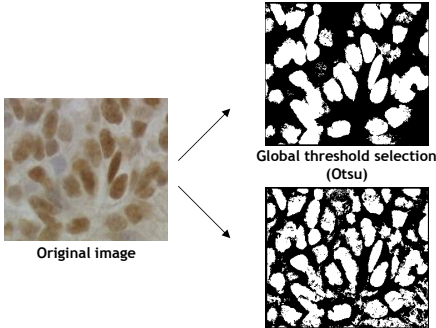


What is the hidden assumption here?

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## Effects



Original image

Global threshold selection (Otsu)

Local threshold selection (Niblack)

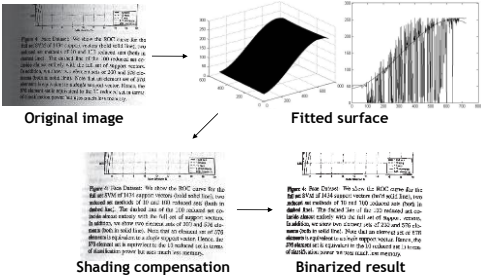
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## Additional Improvements

- Document images often contain a smooth gradient

⇒ Try to fit that gradient with a polynomial function



Original image

Fitted surface

Shading compensation

Binarized result

Source: S. Lu & C. Tan, ICDA'07

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## Polynomial Surface Fitting

- Polynomial surface of degree  $d$

$$f(x, y) = \sum_{i+j=0}^d b_{i,j} x^i y^j$$

- For an image pixel  $(x_0, y_0)$  with intensity  $I_0$ , this means

$$b_{0,0} + b_{1,0}x_0 + b_{0,1}y_0 + b_{2,0}x_0^2 + b_{1,1}x_0y_0 + \dots + b_{0,3}y_0^3 = I_0$$

- Least-squares estimation, e.g. for  $d = 3$

$$\begin{bmatrix} 1 & x_0 & y_0 & x_0^2 & x_0y_0 & \dots & y_0^3 \\ 1 & x_1 & y_1 & x_1^2 & x_1y_1 & \dots & y_1^3 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & y_n & x_n^2 & x_ny_n & \dots & y_n^3 \end{bmatrix} \begin{bmatrix} b_{0,0} \\ b_{1,0} \\ \vdots \\ b_{0,3} \end{bmatrix} = \begin{bmatrix} I_0 \\ I_1 \\ \vdots \\ I_n \end{bmatrix}$$

Solution with pseudo-inverse:

$$b = (A^T A)^{-1} A^T I$$

Matlab (using SVD):

$$b = I \setminus A$$

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## Surface Fitting

- Iterative Algorithm
  - Fit parametric surface to all points in region.
  - Subtract estimated surface.
  - Apply global threshold (e.g. with Otsu method)
  - Fit surface to all background pixels in original region.
  - Subtract estimated surface.
  - Apply global threshold (Otsu)
  - Iterate further if needed...

Initial guess

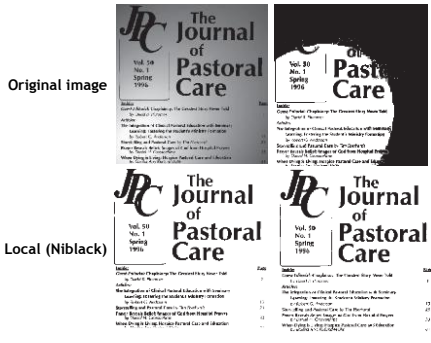
Refined guess

- The first pass also takes foreground pixels into account.
  - This is corrected in the following passes.
  - Basic assumption here: most pixels belong to the background.

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## Result Comparison



Original image

Global (Otsu)

Local (Niblack)

Polynomial + Global

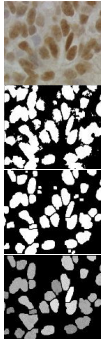
Source: S. Lu & C. Tan, ICDA'07

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## Outline of Today's Lecture

- Convert the image into binary form
  - Thresholding
- Clean up the thresholded image
  - Morphological operators
- Extract individual objects
  - Connected Components Labeling
- Describe the objects
  - Region properties



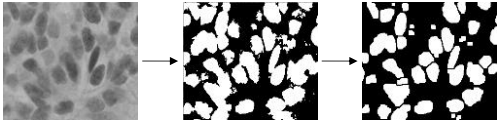
B. Leibe Image Source: D. Kim et al., Cytometry 35(1), 1999 28

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## Cleaning the Binarized Results

- Results of thresholding often still contain noise
  - Remove isolated points and small structures
  - Fill holes
- Necessary cleaning operations
  - Remove isolated points and small structures
  - Fill holes

⇒ Morphological Operators

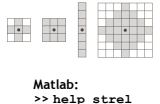


B. Leibe Image Source: D. Kim et al., Cytometry 35(1), 1999 29

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## Morphological Operators

- Basic idea
  - Scan the image with a structuring element
  - Perform set operations (intersection, union) of image content with structuring element
- Two basic operations
  - Dilation (Matlab: imdilate)
  - Erosion (Matlab: imerode)
- Several important combinations
  - Opening (Matlab: imopen)
  - Closing (Matlab: imclose)
  - Boundary extraction



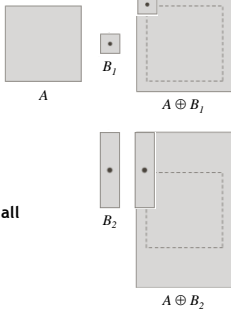
Matlab: >> help strel

B. Leibe Image Source: R.C. Gonzales & R.E. Woods 30

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## Dilation

- Definition
  - "The dilation of  $A$  by  $B$  is the set of all displacements  $z$ , such that  $(B)_z$  and  $A$  overlap by at least one element".
  - $((\hat{B})_z)$  is the mirrored version of  $B$ , shifted by  $z$
- Effects
  - If current pixel  $z$  is foreground, set all pixels under  $(B)_z$  to foreground.
  - ⇒ Expand connected components
  - ⇒ Grow features
  - ⇒ Fill holes

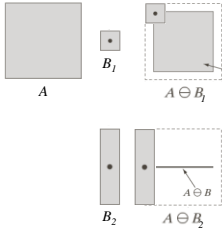


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## Erosion

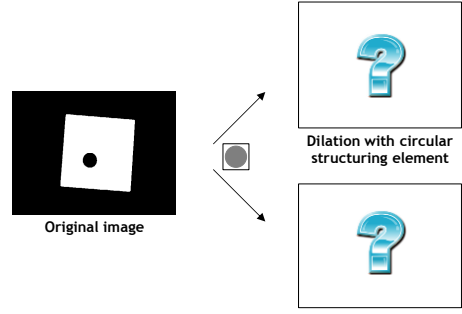
- Definition
  - "The erosion of  $A$  by  $B$  is the set of all displacements  $z$ , such that  $(B)_z$  is entirely contained in  $A$ ".
- Effects
  - If not every pixel under  $(B)_z$  is foreground, set the current pixel  $z$  to background.
  - ⇒ Erode connected components
  - ⇒ Shrink features
  - ⇒ Remove bridges, branches, noise



B. Leibe Image Source: R.C. Gonzales & R.E. Woods 32

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## Effects



Original image

Dilation with circular structuring element

Erosion with circular structuring element

B. Leibe Image Source: <http://homepages.inf.ed.ac.uk/rbf/HIP02/> 33

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## Effects

Original image

Dilation with circular structuring element

Erosion with circular structuring element

B. Leibe Image Source: <http://homepages.inf.ed.ac.uk/rbf/HIP07/>

34

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## Opening

- Definition
  - Sequence of Erosion and Dilation
$$A \circ B = (A \ominus B) \oplus B$$
- Effect
  - $A \circ B$  is defined by the points that are reached if  $B$  is rolled around inside  $A$ .
  - ⇒ Remove small objects, keep original shape.

B. Leibe Image Source: R.C. Gonzales & R.E. Woods

35

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## Effect of Opening

- Feature selection through *size* of structuring element

Original image

Thresholded

Opening with small structuring element

Opening with larger structuring element

B. Leibe Image Source: <http://homepages.inf.ed.ac.uk/rbf/HIP07/>

36

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## Effect of Opening

- Feature selection through *shape* of structuring element

Input Image

Opening with circular structuring element

- How could we have extracted the lines?

B. Leibe Image Source: <http://homepages.inf.ed.ac.uk/rbf/HIP07/>

37

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## Closing

- Definition
  - Sequence of Dilation and Erosion
$$A \cdot B = (A \oplus B) \ominus B$$
- Effect
  - $A \cdot B$  is defined by the points that are reached if  $B$  is rolled around on the outside of  $A$ .
  - ⇒ Fill holes, keep original shape.

B. Leibe Image Source: R.C. Gonzales & R.E. Woods

38

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## Effect of Closing

- Fill holes in thresholded image (e.g. due to specularities)

Original image

Thresholded

Closing with circular structuring element

Size of structuring element determines which structures are selectively filled.

B. Leibe Image Source: <http://homepages.inf.ed.ac.uk/rbf/HIP07/>

39



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### Example Application: Opening + Closing

Original image    Opening    Closing

Erosion    Dilation    Dilation    Erosion

Structuring element

Eroded image    Dilated image

B. Leibe    Source: R.C. Gonzales & R.E. Woods

40

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### Application: Blob Tracking

Slide credit: K. Grauman

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Slide credit: K. Grauman

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### Morphological Boundary Extraction

- Definition
  - First erode  $A$  by  $B$ , then subtract the result from the original  $A$ .
  - $\beta(A) = A - (A \ominus B)$
- Effects
  - If a  $3 \times 3$  structuring element is used, this results in a boundary that is exactly 1 pixel thick.

B. Leibe    Source: R.C. Gonzales & R.E. Woods

44

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### Morphology Operators on Grayscale Images

- Sidenote
  - Dilation and erosion are typically performed on binary images.
  - If image is grayscale: for dilation take the neighborhood max, for erosion take the min.

Original    Dilated    Eroded

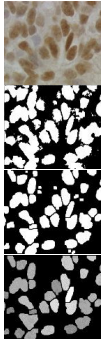
Slide credit: Kristen Grauman

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45

**Outline of Today's Lecture**

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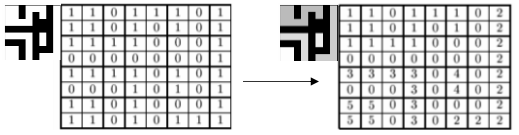
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Image Source: D. Kim et al., Cytometry 35(1), 1997

46

**Connected Components Labeling**

- Goal: Identify distinct regions



Binary image

Connected components labeling

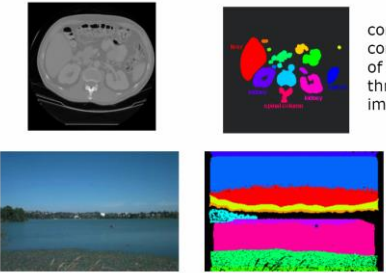
Sources: Shapiro & Stockman, Chandra

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47

**Connected Components Examples**



connected components of 1's from thresholded image

connected components of cluster labels

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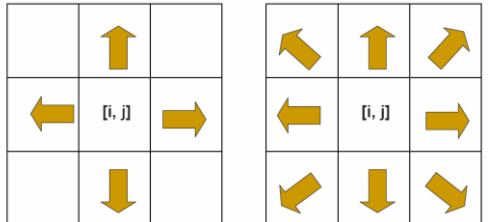
Source: Pinar Duygulu

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48

**Connectedness**

- Which pixels are considered neighbors?



4-connected

8-connected

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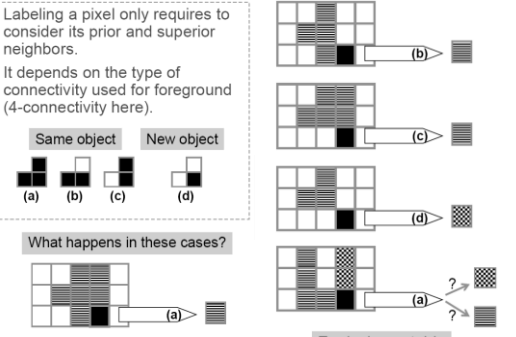
Source: Chaitanya Chandra

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49

**Sequential Connected Components**

- Labeling a pixel only requires to consider its prior and superior neighbors.
- It depends on the type of connectivity used for foreground (4-connectivity here).



Same object

New object

(a) (b) (c) (d)

What happens in these cases?

Equivalence table

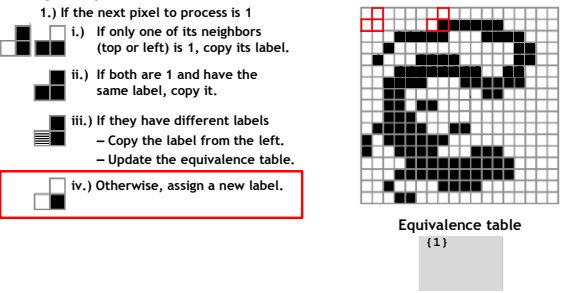
Slide credit: J. Neira

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50

**Sequential Connected Components (2)**

- Process the image from left to right, top to bottom:
  - If the next pixel to process is 1
    - If only one of its neighbors (top or left) is 1, copy its label.
    - If both are 1 and have the same label, copy it.
    - If they have different labels
      - Copy the label from the left.
      - Update the equivalence table.
    - Otherwise, assign a new label.



Equivalence table

{1}

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Slide credit: J. Neira

B. Leibe

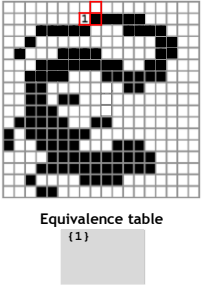
51



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## Sequential Connected Components (2)

- Process the image from left to right, top to bottom:
  - If the next pixel to process is 1
    - If only one of its neighbors (top or left) is 1, copy its label.
    - If both are 1 and have the same label, copy it.
    - If they have different labels
      - Copy the label from the left.
      - Update the equivalence table.
    - Otherwise, assign a new label.



Equivalence table  
{1}

52

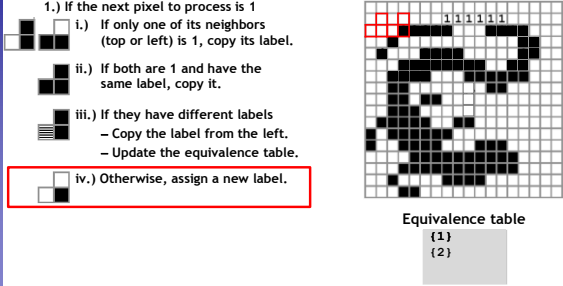
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Slide credit: J. Neira      B. Leibe

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Equivalence table  
{1}  
{2}

53

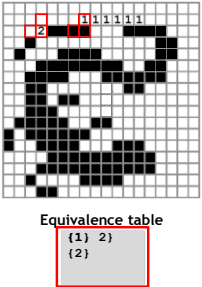
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Equivalence table  
{1}, {2}  
{2}

54

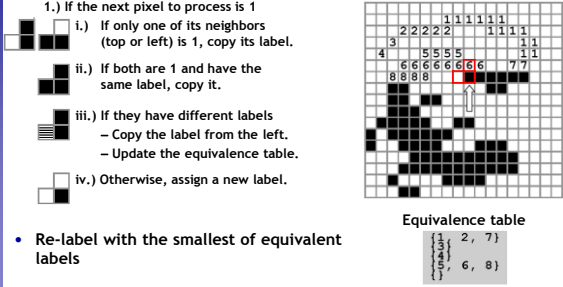
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      - Update the equivalence table.
    - Otherwise, assign a new label.
- Re-label with the smallest of equivalent labels



Equivalence table  
{2}, {7}  
{6}, {8}

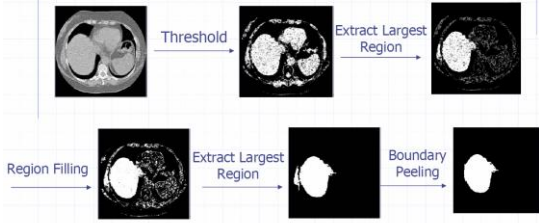
55

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## Application: Segmentation of a Liver



*Application by Jie Zhu, Cornell University*

56

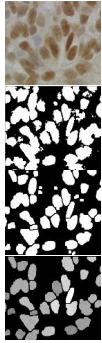
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Slide credit: Li Shen      B. Leibe

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## Outline of Today's Lecture

- Convert the image into binary form
  - Thresholding
- Clean up the thresholded image
  - Morphological operators
- Extract individual objects
  - Connected Components Labeling
- Describe the objects
  - Region properties



57

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Image Source: D. Kim et al., Cytometry 35(1), 1999

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## Region Properties

- From the previous steps, we can obtain separated objects.
- Some useful features can be extracted once we have connected components, including
  - Area
  - Centroid
  - Extremal points, bounding box
  - Circularity
  - Spatial moments

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## Area and Centroid

- We denote the set of pixels in a region by  $R$
- Assuming square pixels, we obtain
  - Area: 
$$A = \sum_{(x,y) \in R} 1$$
  - Centroid: 
$$\bar{x} = \frac{1}{A} \sum_{(x,y) \in R} x$$
  

$$\bar{y} = \frac{1}{A} \sum_{(x,y) \in R} y$$

Source: Shapiro & Stockman

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59

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## Circularity

- Measure the deviation from a perfect circle
  - Circularity: 
$$C = \frac{\mu_R}{\sigma_R}$$

where  $\mu_R$  and  $\sigma_R^2$  are the mean and variance of the distance from the centroid of the shape to the boundary pixels  $(x_k, y_k)$ .

Mean radial distance: 
$$\mu_R = \frac{1}{K} \sum_{k=0}^{K-1} \|(x_k, y_k) - (\bar{x}, \bar{y})\|$$

Variance of radial distance: 
$$\sigma_R^2 = \frac{1}{K} \sum_{k=0}^{K-1} \left[ \|(x_k, y_k) - (\bar{x}, \bar{y})\| - \mu_R \right]^2$$

60

Source: Shapiro & Stockman

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## Invariant Descriptors

- Often, we want features independent of location, orientation, scale.

61

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## Central Moments

- $S$  is a subset of pixels (region).
- Central  $(j,k)$ th moment defined as: 
$$\mu_{jk} = \sum_{(x,y) \in S} (x - \bar{x})^j (y - \bar{y})^k$$
- Invariant to translation of  $S$ .
- Interpretation:
  - 0th central moment: area
  - 2nd central moment: variance
  - 3rd central moment: skewness
  - 4th central moment: kurtosis

62

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## Moment Invariants (“Hu Moments”)

- Normalized central moments
 
$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^{\gamma}}, \quad \gamma = \frac{p+q}{2} + 1$$
- From those, a set of *invariant moments* can be defined for object description.
 
$$\phi_1 = \eta_{20} + \eta_{02}$$

$$\phi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2$$

$$\phi_3 = (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2$$

$$\phi_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2$$
- Robust to translation, rotation & scaling, but don't expect wonders (still summary statistics).

63

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## Moment Invariants

$$\phi_5 = (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12}) \left[ (\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2 \right] + (3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03}) \left[ 3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2 \right]$$

$$\phi_6 = (\eta_{20} - \eta_{02}) \left[ (\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2 \right] + 4\eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03})$$

$$\phi_7 = (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12}) \left[ (\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2 \right] + (3\eta_{12} - \eta_{30})(\eta_{21} + \eta_{03}) \left[ 3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2 \right]$$

Often better to use  $\log_{10}(\phi_i)$  instead of  $\phi_i$  directly...

64

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## Axis of Least Second Moment

- Invariance to orientation?
  - Need a common alignment

Axis for which the squared distance to 2D object points is **minimized** (maximized).

- Compute Eigenvectors of 2<sup>nd</sup> moment matrix (Matlab: eig(A))

$$\begin{bmatrix} \mu_{20} & \mu_{11} \\ \mu_{11} & \mu_{02} \end{bmatrix} = VDV^T = \begin{bmatrix} v_{11} & v_{12} \\ v_{22} & v_{22} \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix}^T$$

65

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## Summary: Binary Image Processing

- Pros
  - Fast to compute, easy to store
  - Simple processing techniques
  - Can be very useful for constrained scenarios
- Cons
  - Hard to get "clean" silhouettes
  - Noise is common in realistic scenarios
  - Can be too coarse a representation
  - Cannot deal with 3D changes

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66

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## References and Further Reading

- More on morphological operators can be found in
  - R.C. Gonzales, R.E. Woods, *Digital Image Processing*. Prentice Hall, 2001
- Online tutorial and Java demos available on
  - <http://homepages.inf.ed.ac.uk/rbf/HIPR2/>

67

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## Questions ?

68

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## Demo "Haribo Classification"

69

## You Can Do It At Home...

### Accessing a webcam in Matlab:

```
function out = webcam
% uses "Image Acquisition Toolbox",
adaptorName = 'winvideo';
vidFormat = 'I420_320x240';
vidObj1= videoinput(adaptorName, 1, vidFormat);
set(vidObj1, 'ReturnedColorSpace', 'rgb');
set(vidObj1, 'FramesPerTrigger', 1);
out = vidObj1 ;
```

```
cam = webcam();
img=getsnapshot(cam);
```



*Questions ?*