Computer Vision – Lecture 4

Gradients & Edges

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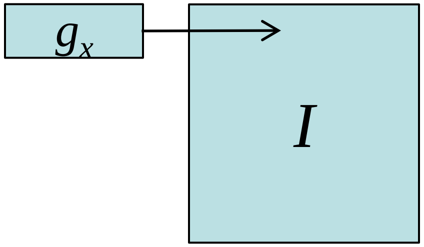
Course Outline

- Image Processing Basics
  - Image Formation
  - Binary Image Processing
  - Linear Filters
  - Edge & Structure Extraction
- Segmentation
- Local Features & Matching
- Object Recognition and Categorization
- 3D Reconstruction
- Motion and Tracking

Topics of This Lecture

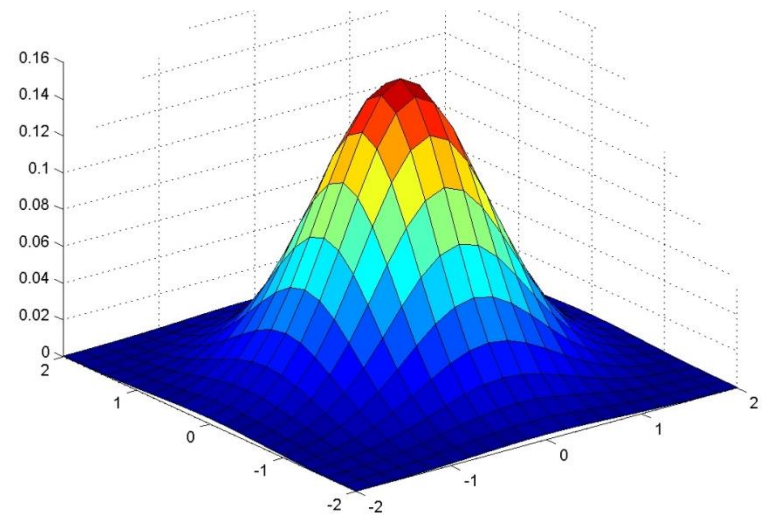
- Recap: Linear Filters
- Multi-Scale representations
  - How to properly rescale an image?
- Filters as templates
  - Correlation as template matching
- Image gradients
  - Derivatives of Gaussian
- Edge detection
  - Canny edge detector



Recap: Gaussian Smoothing

- Gaussian kernel

$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$



- Rotationally symmetric
- Weights nearby pixels more than distant ones
  - This makes sense as 'probabilistic' inference about the signal
- A Gaussian gives a good model of a fuzzy blob

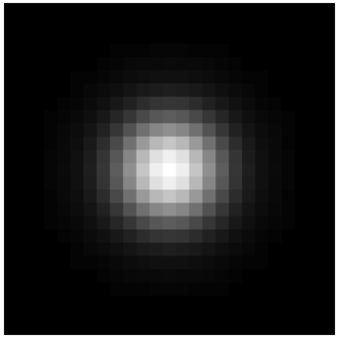
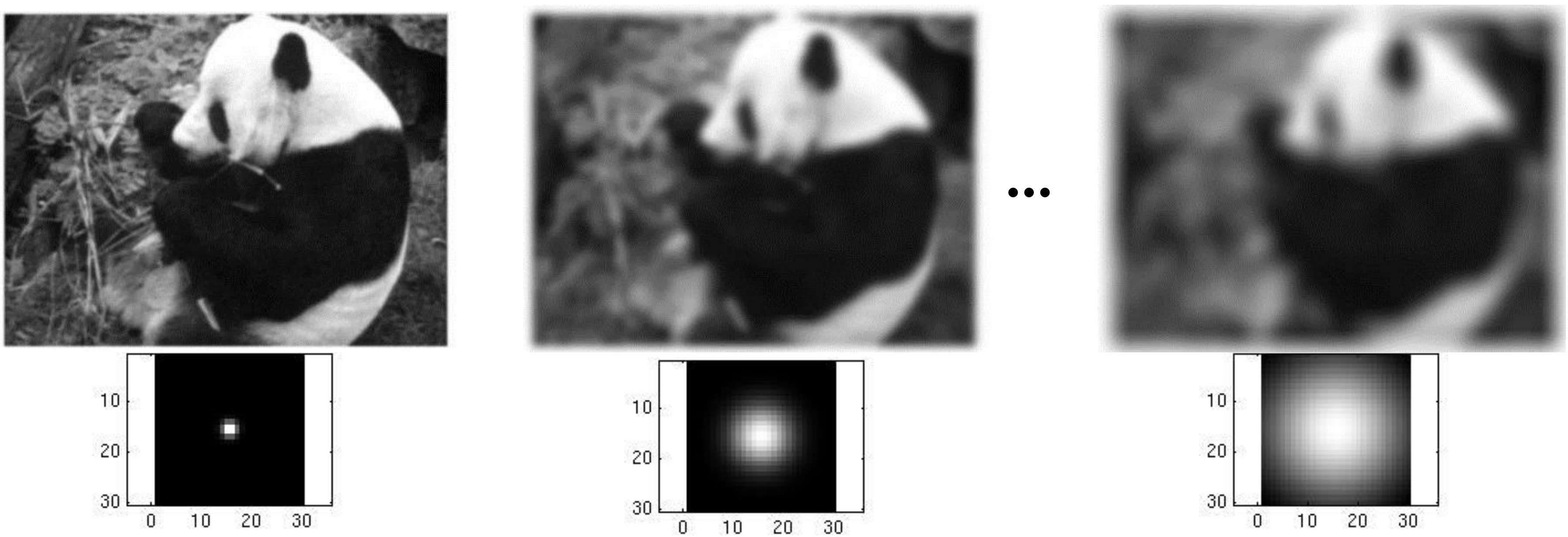


Image Source: Forsyth & Ponce

Recap: Smoothing with a Gaussian

- Parameter σ is the "scale" / "width" / "spread" of the Gaussian kernel and controls the amount of smoothing.

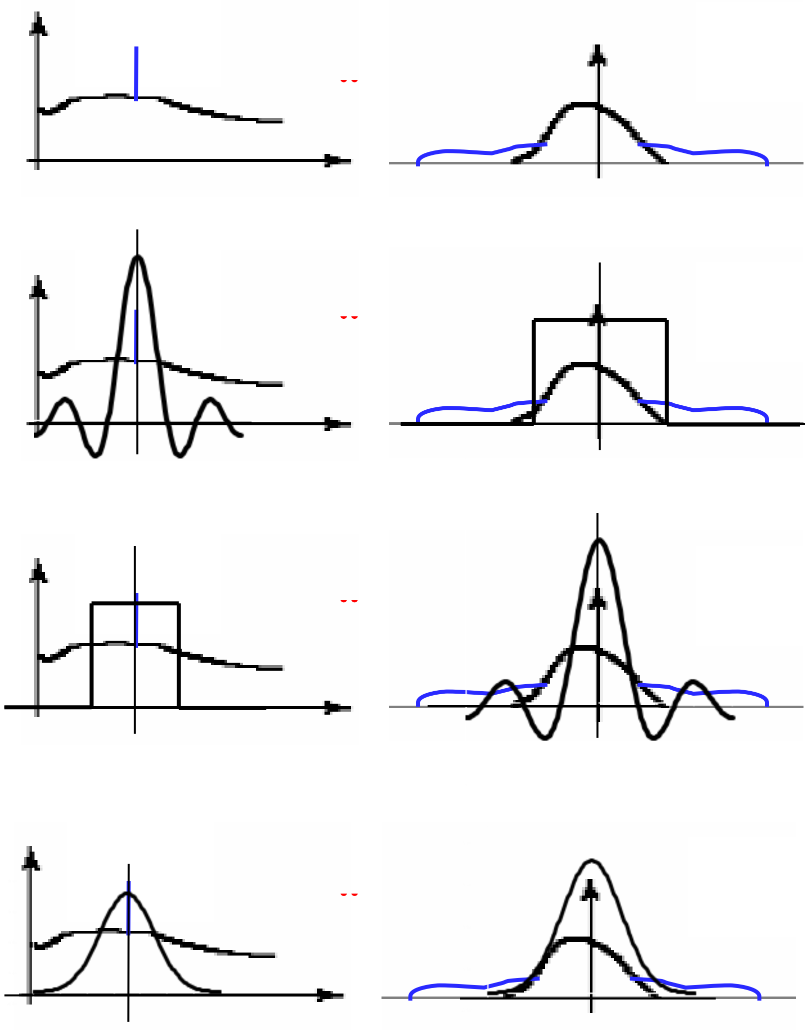



```
for sigma=1:3:10
   h = fspecial('gaussian`, fsize, sigma);
   out = imfilter(im, h);
   imshow(out);
   pause;
end
```

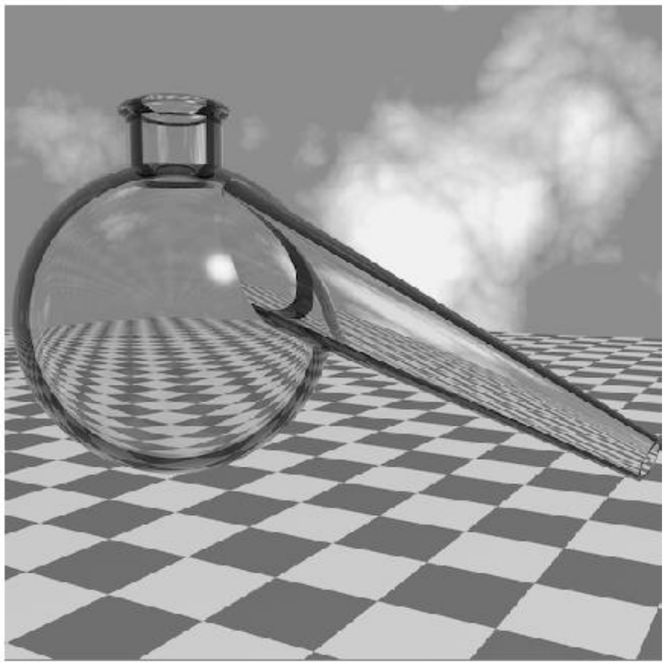
Slide credit: Kristen Grauman

Recap: Effect of Filtering

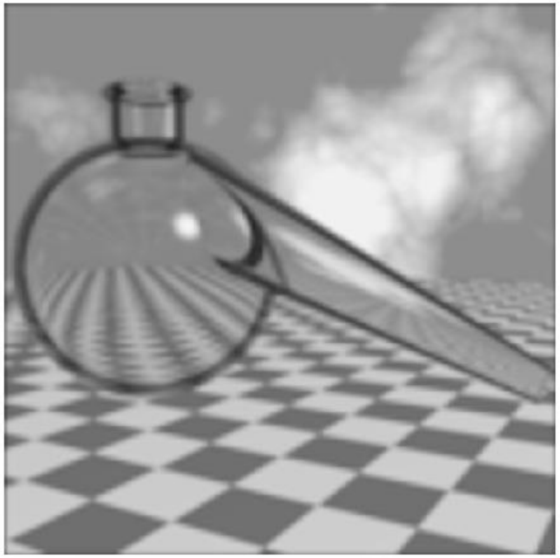
- Noise introduces high frequencies. To remove them, we want to apply a "low-pass" filter.
- The ideal filter shape in the frequency domain would be a box. But this transfers to a spatial sinc, which has infinite spatial support.
- A compact spatial box filter transfers to a frequency sinc, which creates artifacts.
- A Gaussian has compact support in both domains. This makes it a convenient choice for a low-pass filter.



Recap: Low-Pass vs. High-Pass



Original image



Low-pass filtered

High-pass filtered

Image Source: S. Chenney

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Motivation: Fast Search Across Scales

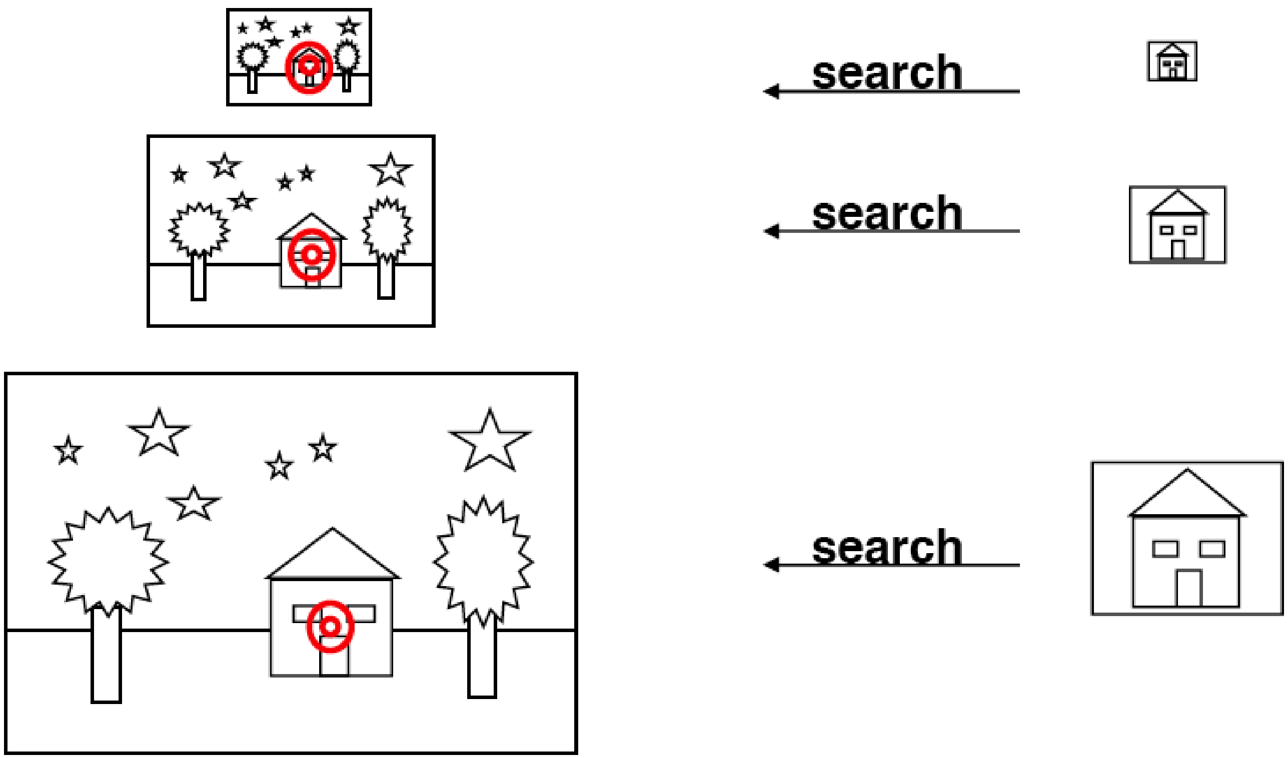


Image Source: Irani & Basri

Recap: Sampling and Aliasing

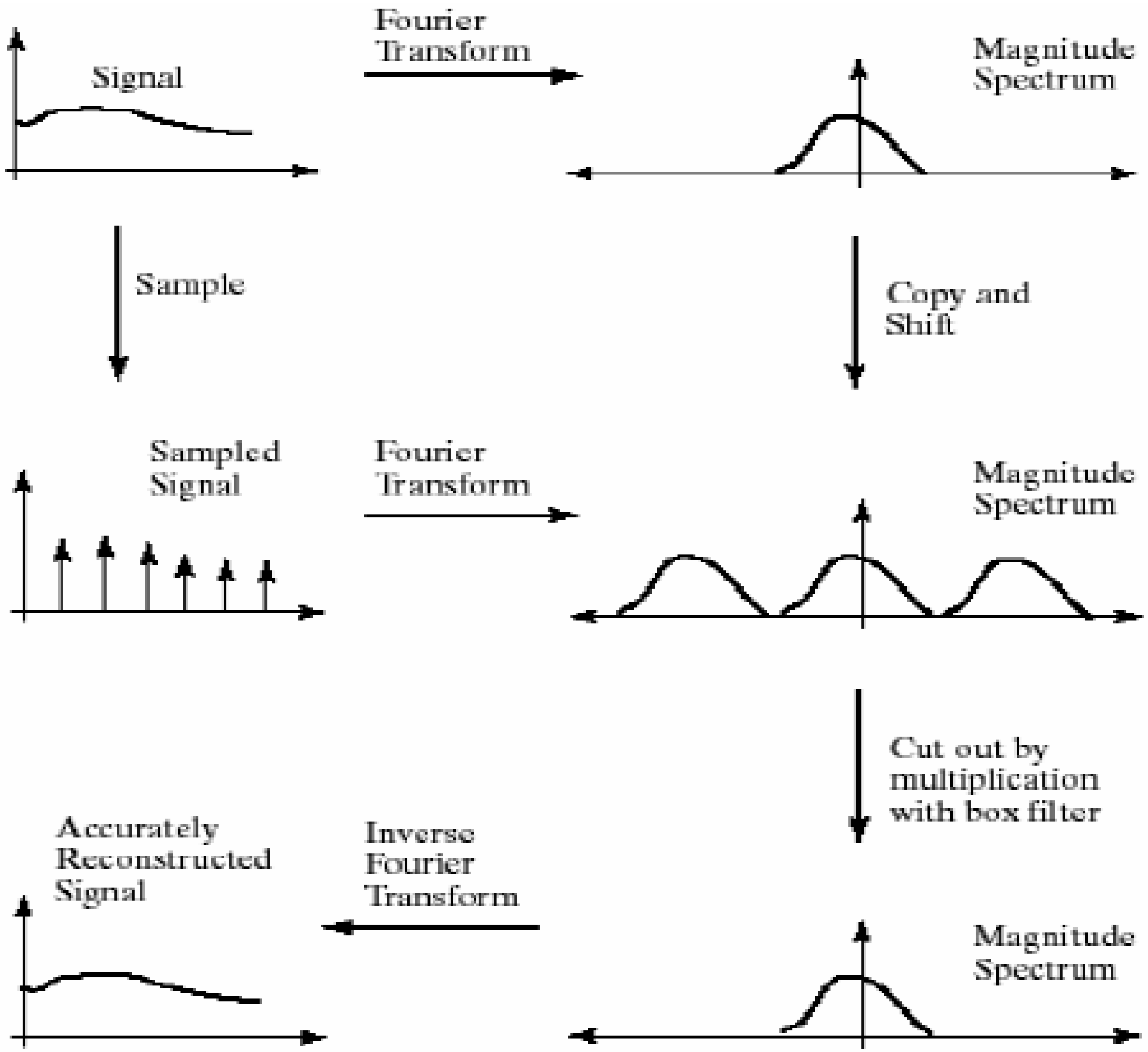


Image Source: Forsyth & Ponce

Recap: Sampling and Aliasing

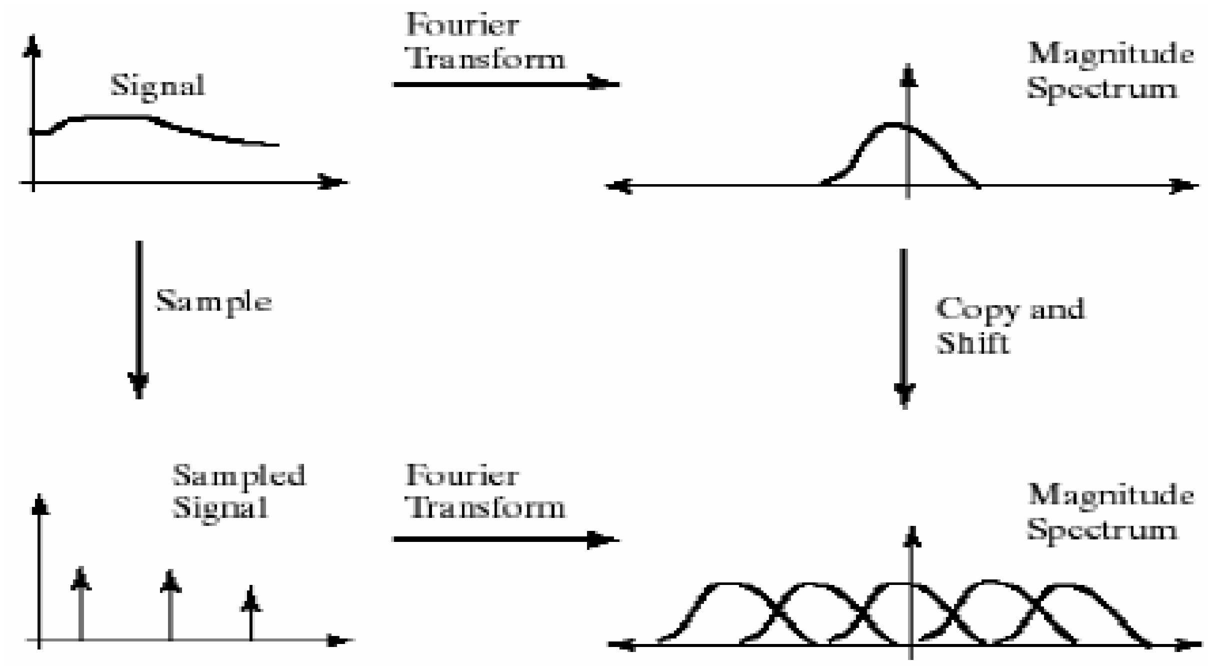


Image Source: Forsyth & Ponce

Recap: Sampling and Aliasing

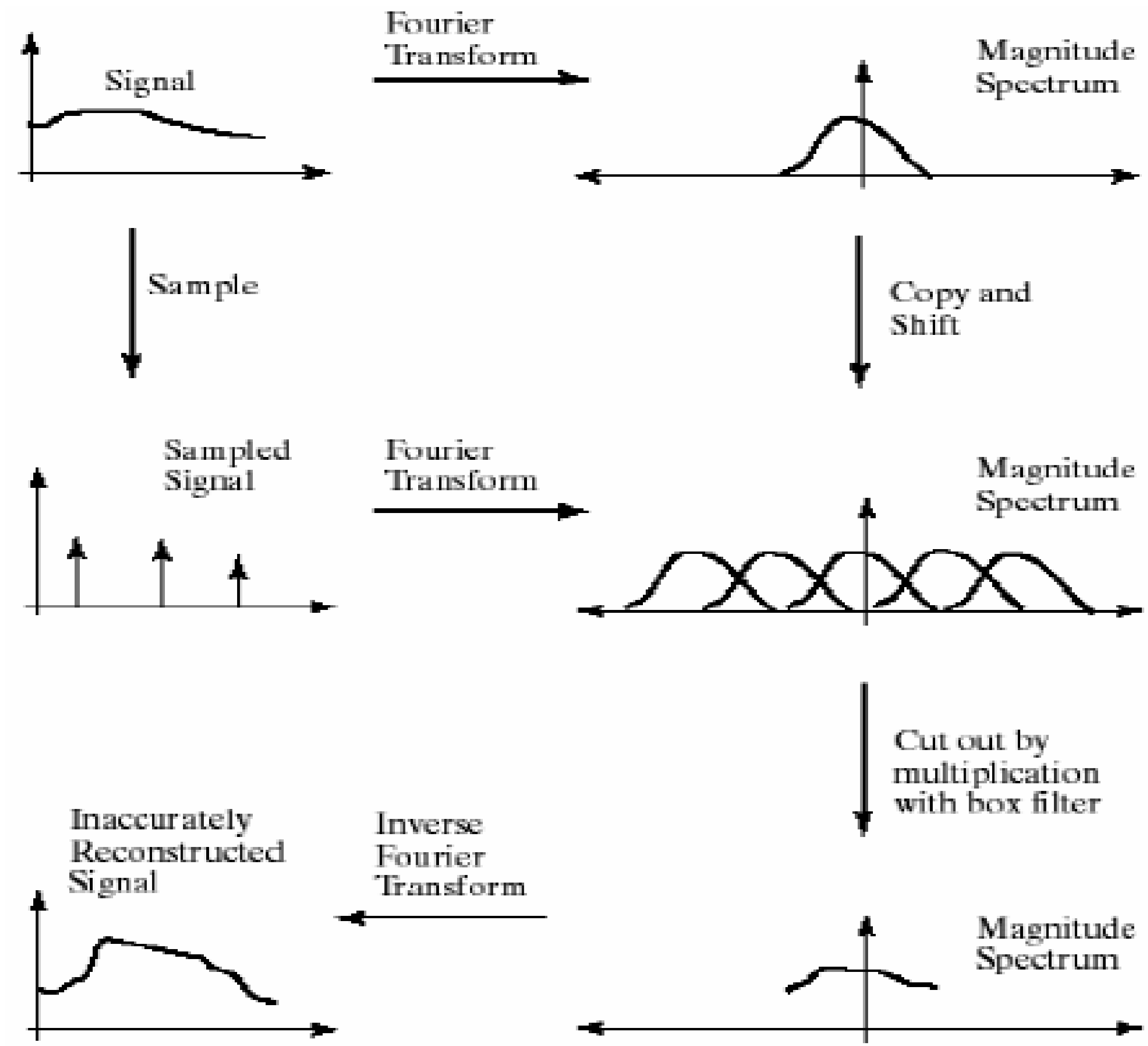
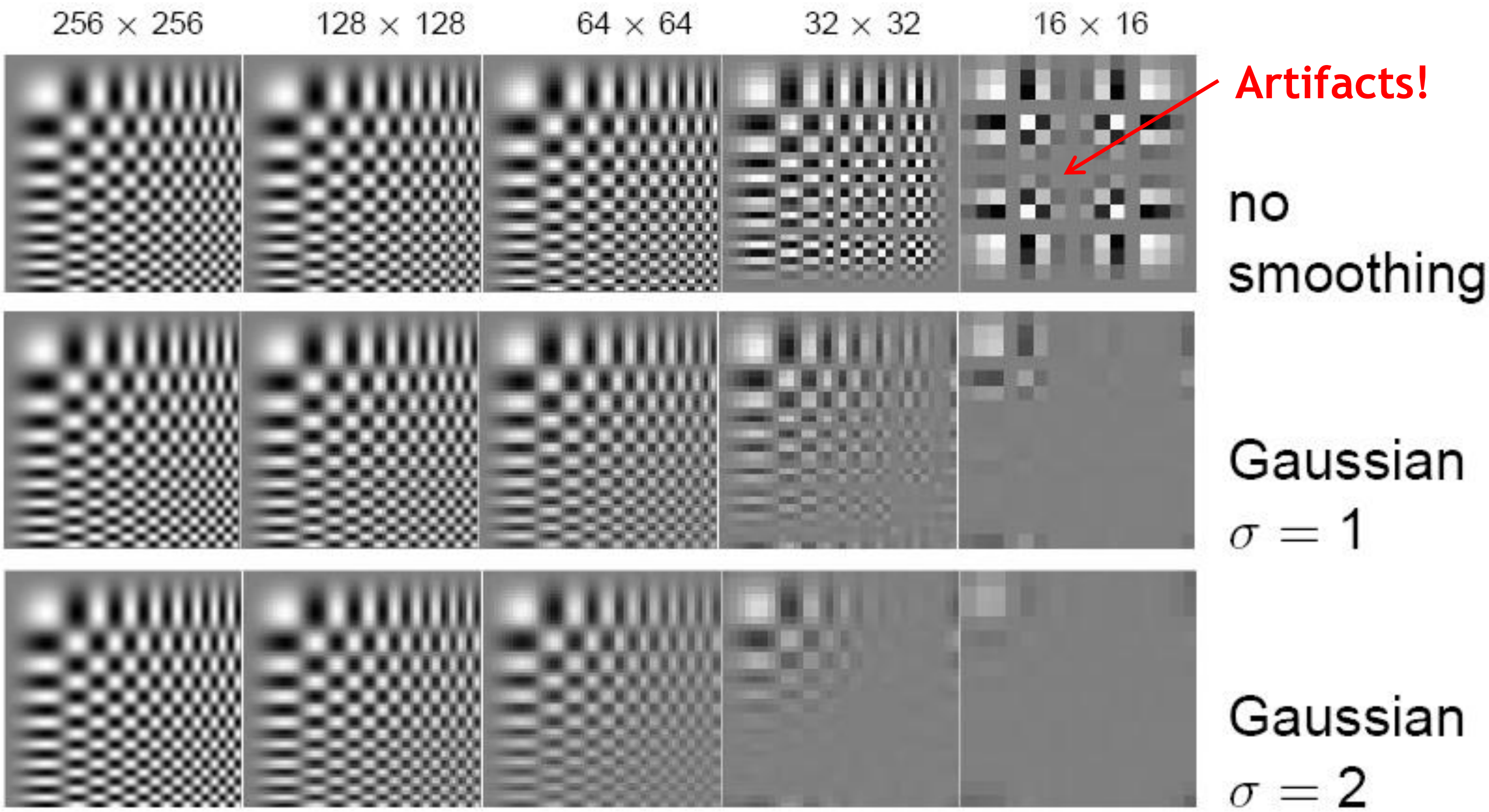


Image Source: Forsyth & Ponce

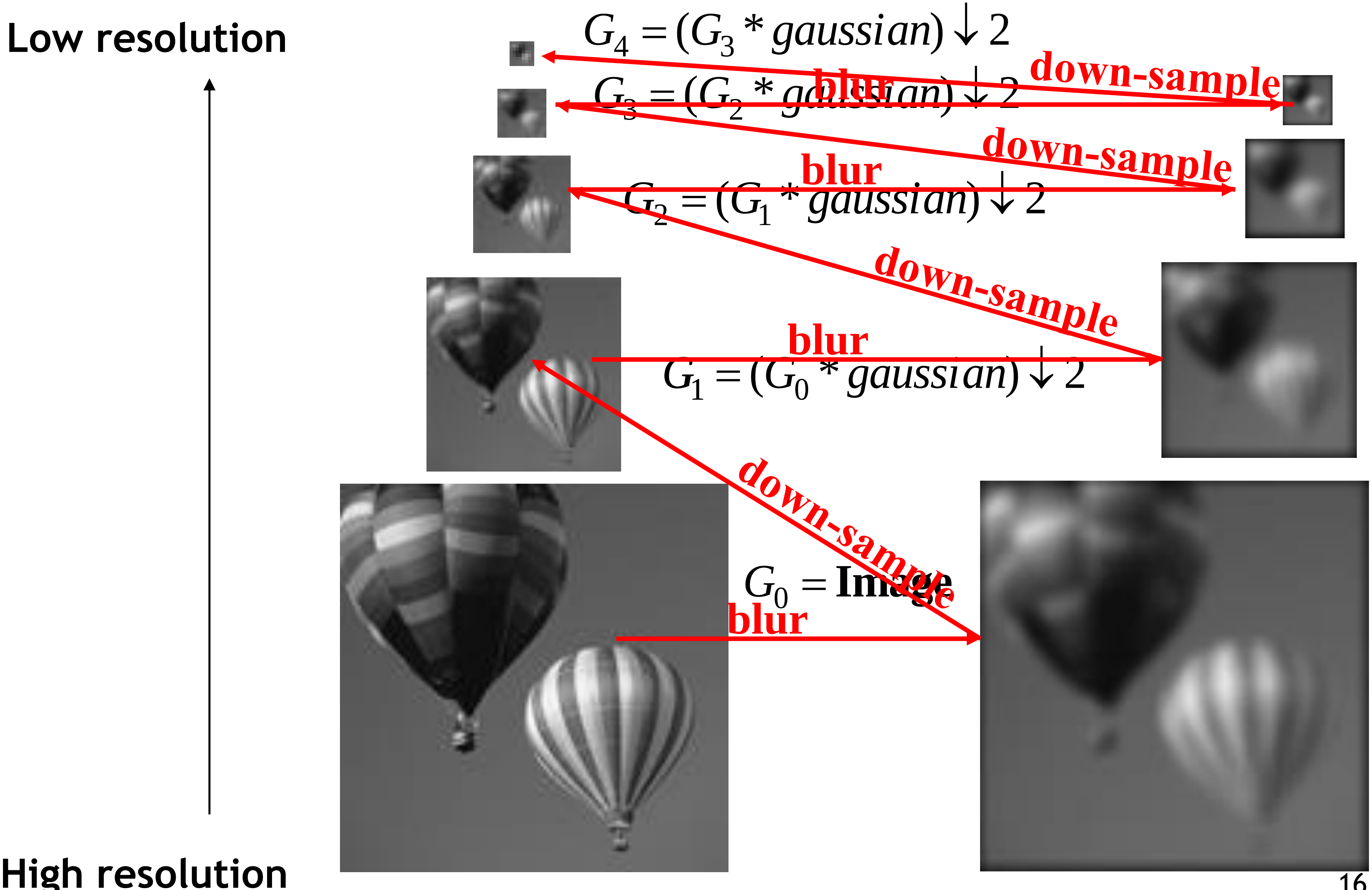
Recap: Resampling with Prior Smoothing



- **Note: We cannot recover the high frequencies, but we can avoid artifacts by smoothing before resampling.**

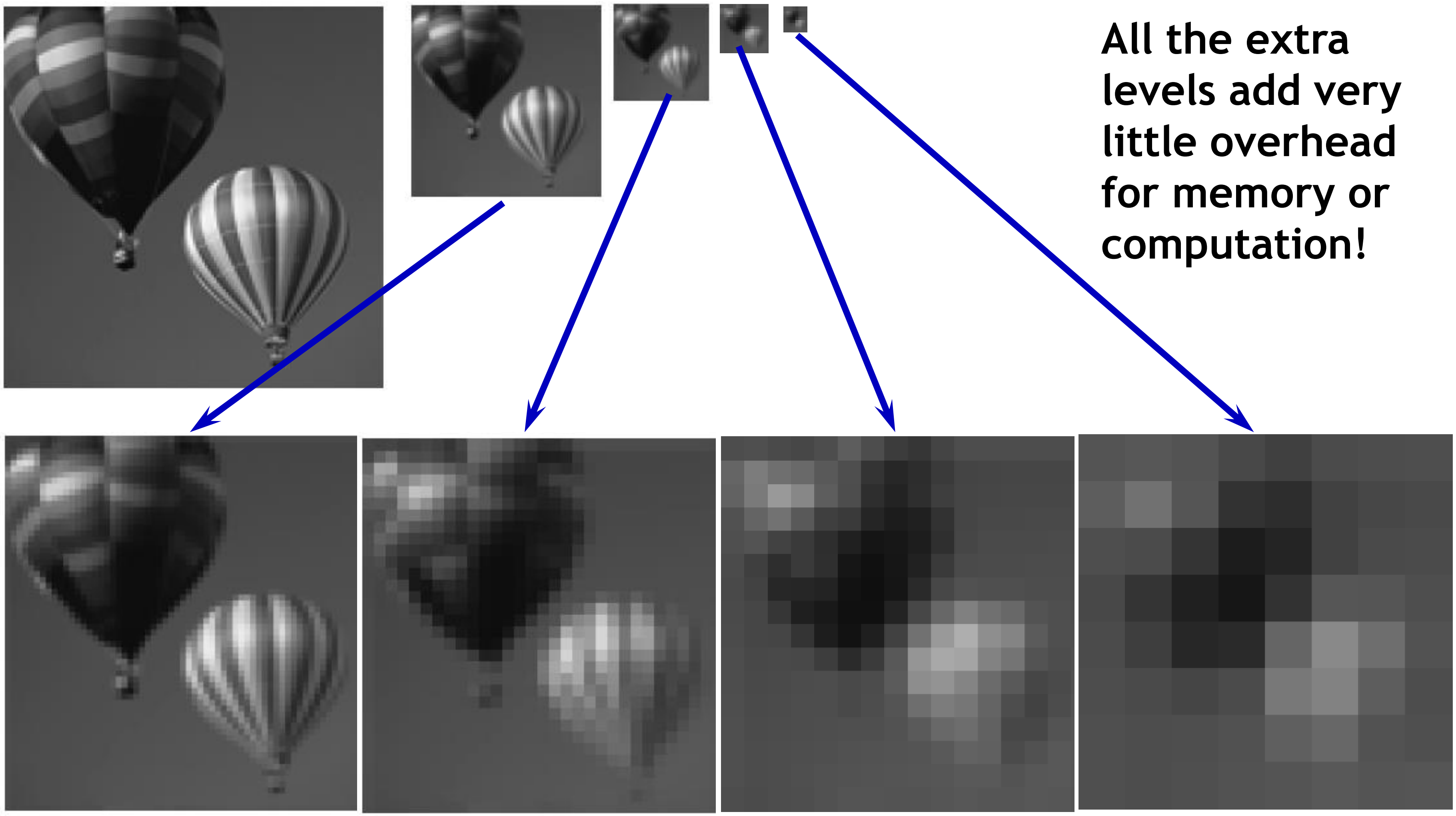
Image Source: Forsyth & Ponce

The Gaussian Pyramid



Source: Irani & Basri

Gaussian Pyramid – Stored Information



All the extra levels add very little overhead for memory or computation!

Source: Irani & Basri

Summary: Gaussian Pyramid

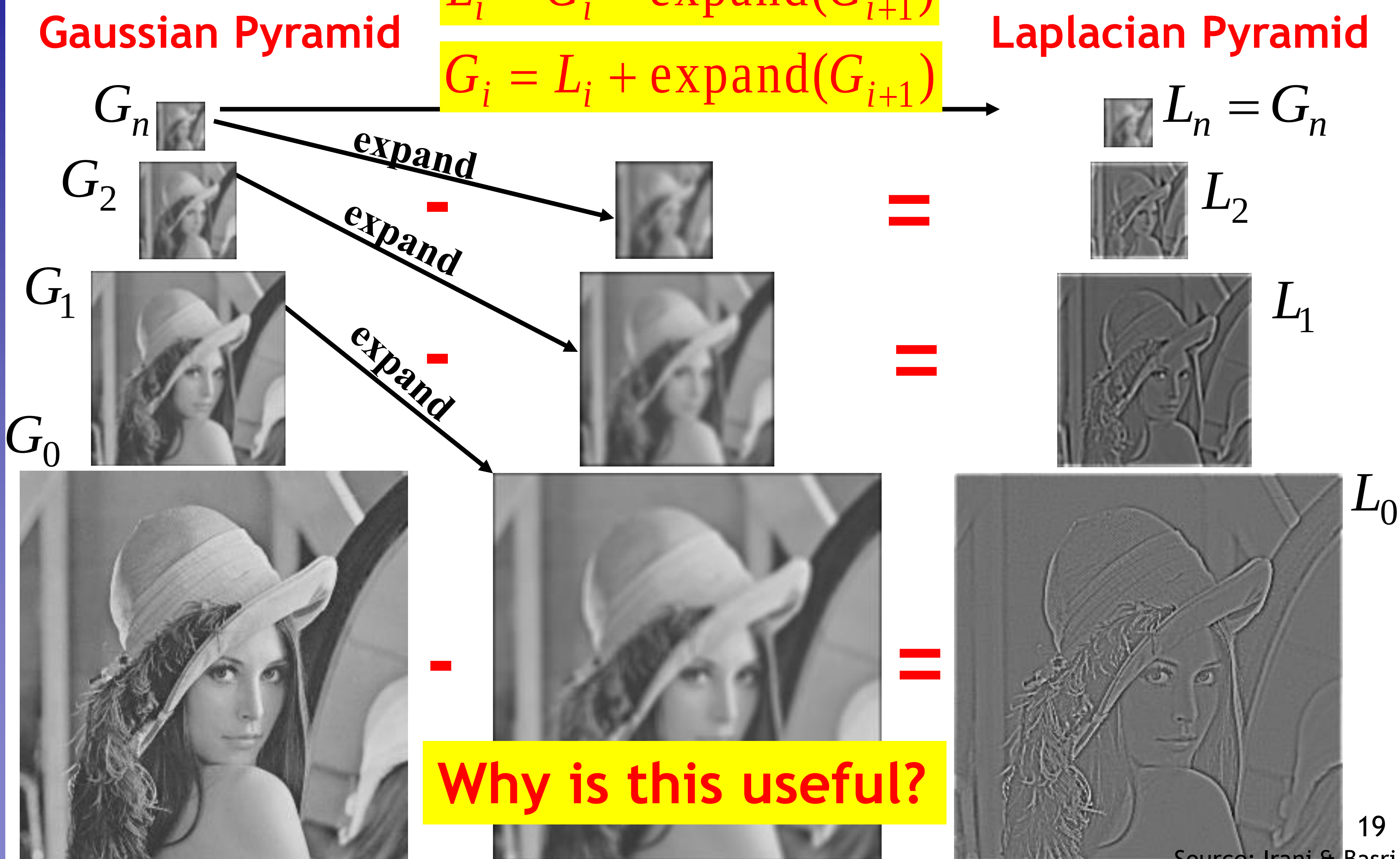
- Construction: create each level from previous one
  - Smooth and sample
- Smooth with Gaussians, in part because
  - a Gaussian*Gaussian = another Gaussian
  - $G(\sigma_1) * G(\sigma_2) = G(\mathrm{sqrt}(\sigma_1{}^2 + \sigma_2{}^2))$
- Gaussians are low-pass filters, so the representation is redundant once smoothing has been performed.

  $\Rightarrow$ There is no need to store smoothed images at the full original resolution.

The Laplacian Pyramid

$$L_i = G_i - \text{expand}(G_{i+1})$$

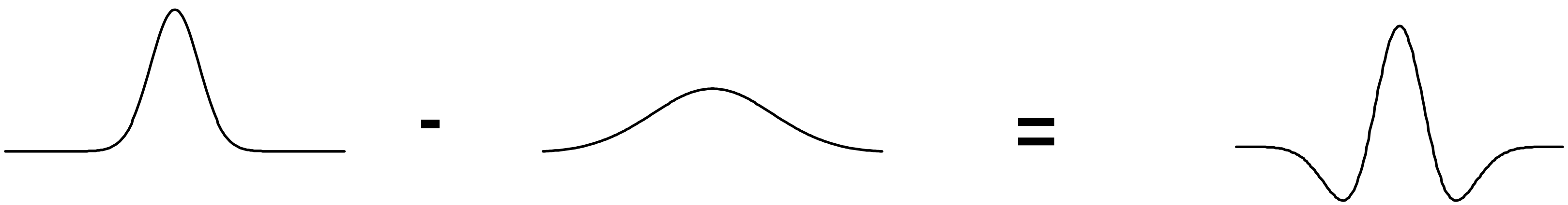
$$G_i = L_i + \text{expand}(G_{i+1})$$



Why is this useful?

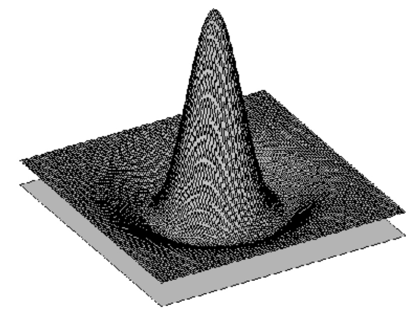
Source: Irani & Basri

Laplacian ~ Difference of Gaussian



DoG = Difference of Gaussians

Cheap approximation - no derivatives needed.



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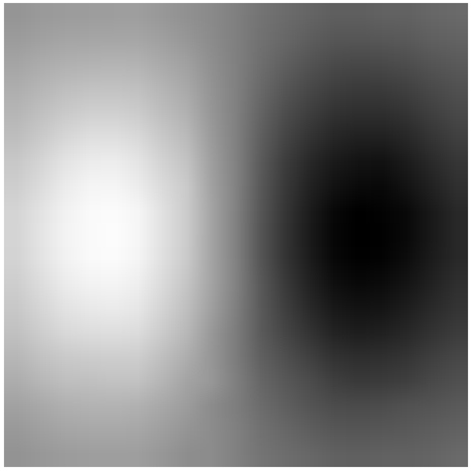
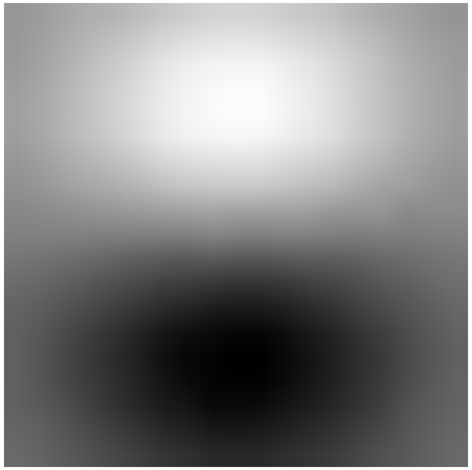
Note: Filters are Templates

- Applying a filter at some point can be seen as taking a dot-product between the image and some vector.
- Filtering the image is a set of dot products.

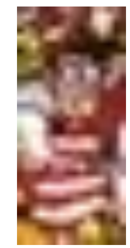
- Insight
  - Filters look like the effects they are intended to find.
  - Filters find effects they look like.





Where's Waldo?

Scene

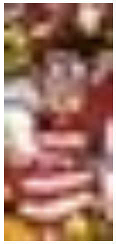


Template

Slide credit: Kristen Grauman

Where's Waldo?

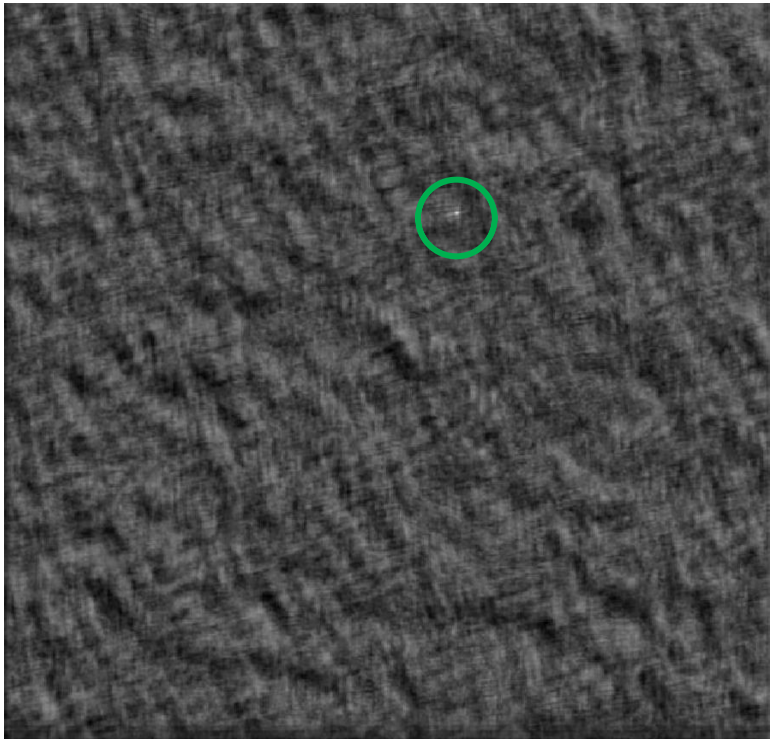
Detected template



Template

Where's Waldo?

Detected template



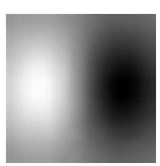
Correlation map

Correlation as Template Matching

- **Think of filters as a dot product of the filter vector with the image region**
  - **Now measure the angle between the vectors**

$$a \cdot b = |a||b| \cos\theta \qquad \cos\theta = \frac{a \cdot b}{|a||b|}$$

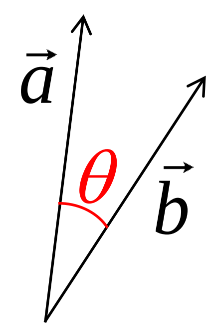
  - **Angle (similarity) between vectors can be measured by normalizing the length of each vector to 1 and taking the dot product.**



Template

·

Image region



Vector interpretation

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- **Image gradients**
  - **Derivatives of Gaussian**
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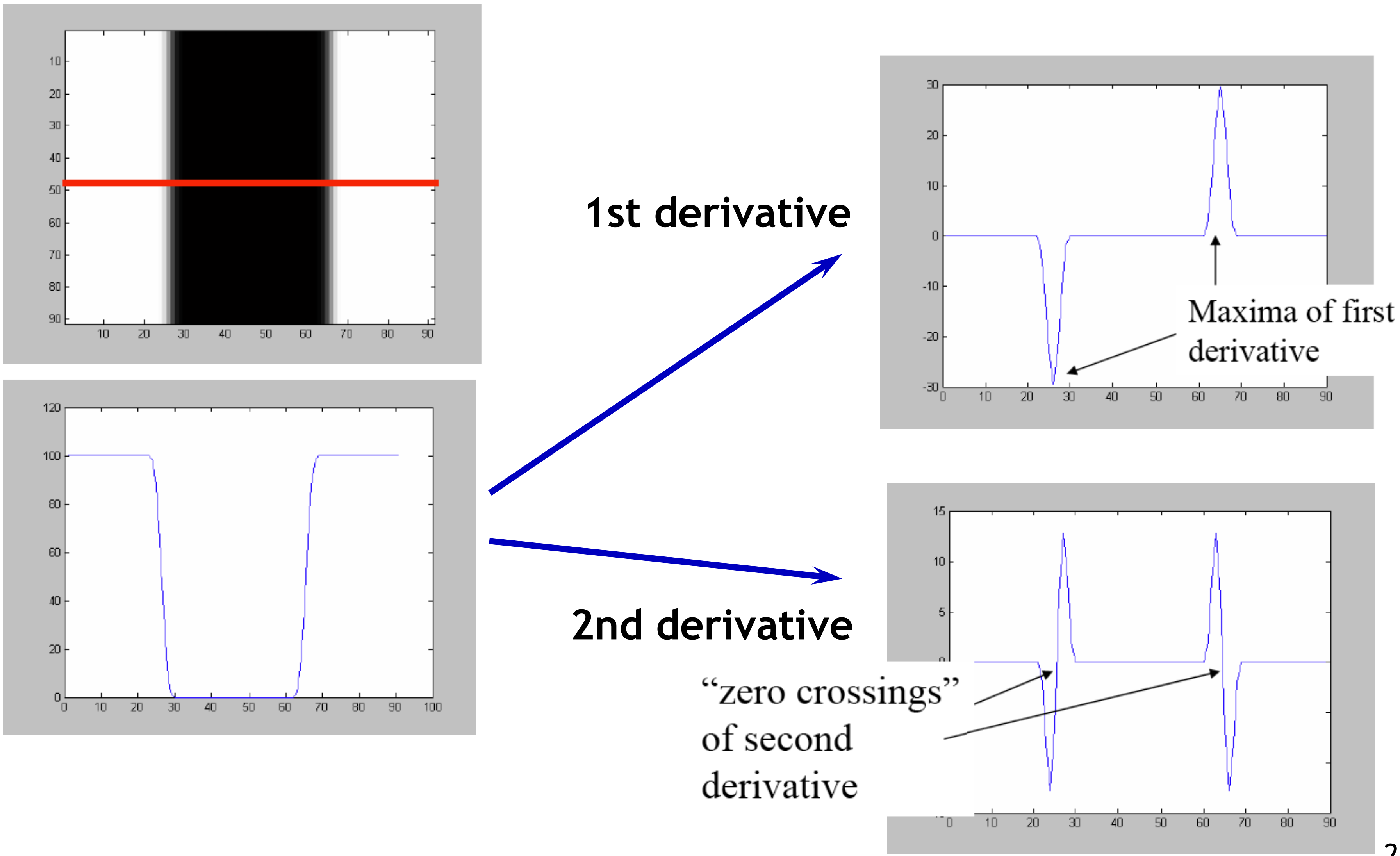
Derivatives and Edges…



Differentiation and Convolution

- For the 2D function $f(x,y)$, the partial derivative is:

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\varepsilon \to 0} \frac{f(x+\varepsilon, y) - f(x,y)}{\varepsilon}$$

- For discrete data, we can approximate this using finite differences:

$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f(x+1, y) - f(x,y)}{1}$$

- To implement the above as convolution, what would be the associated filter?

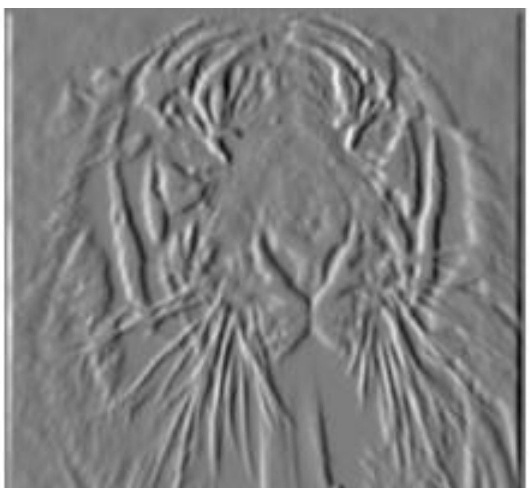
1	-1

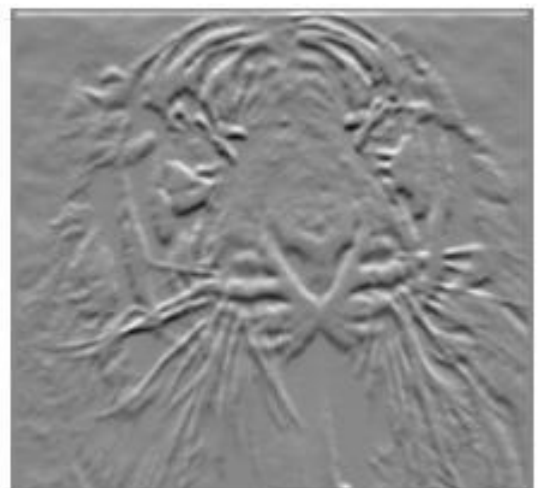
Partial Derivatives of an Image

$$\frac{\partial f(x,y)}{\partial x}$$

$$\frac{\partial f(x,y)}{\partial y}$$

-1	1





-1
1

? or

1
-1

Which shows changes with respect to x?

Assorted Finite Difference Filters

Prewitt: $M_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$; $M_y = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$

Sobel: $M_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$; $M_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$

Roberts: $M_x = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$; $M_y = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

```
>> My = fspecial('sobel');
>> outim = imfilter(double(im), My);
>> imagesc(outim);
>> colormap gray;
```

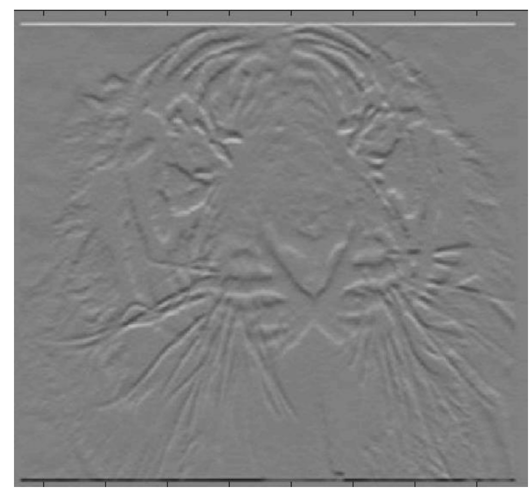
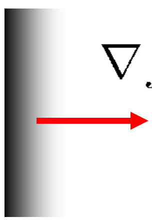


Image Gradient

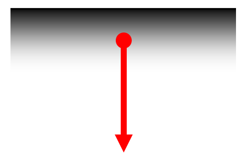
- **The gradient of an image:**

$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

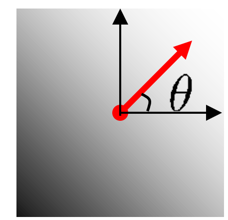
- **The gradient points in the direction of most rapid intensity change**



$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$



$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$



$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

- **The gradient direction (orientation of edge normal) is given by:**

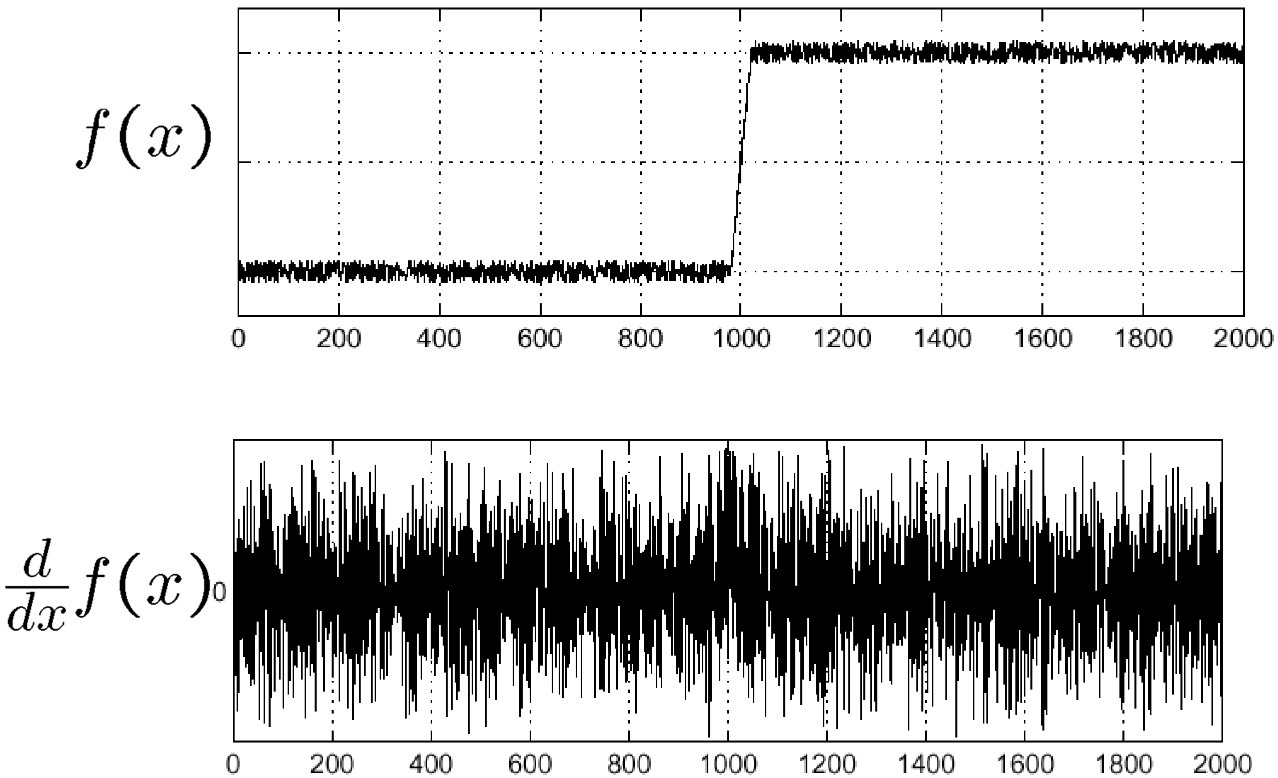
$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

- **The edge strength is given by the gradient magnitude**

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Effect of Noise

- Consider a single row or column of the image
  - Plotting intensity as a function of position gives a signal



Where is the edge?

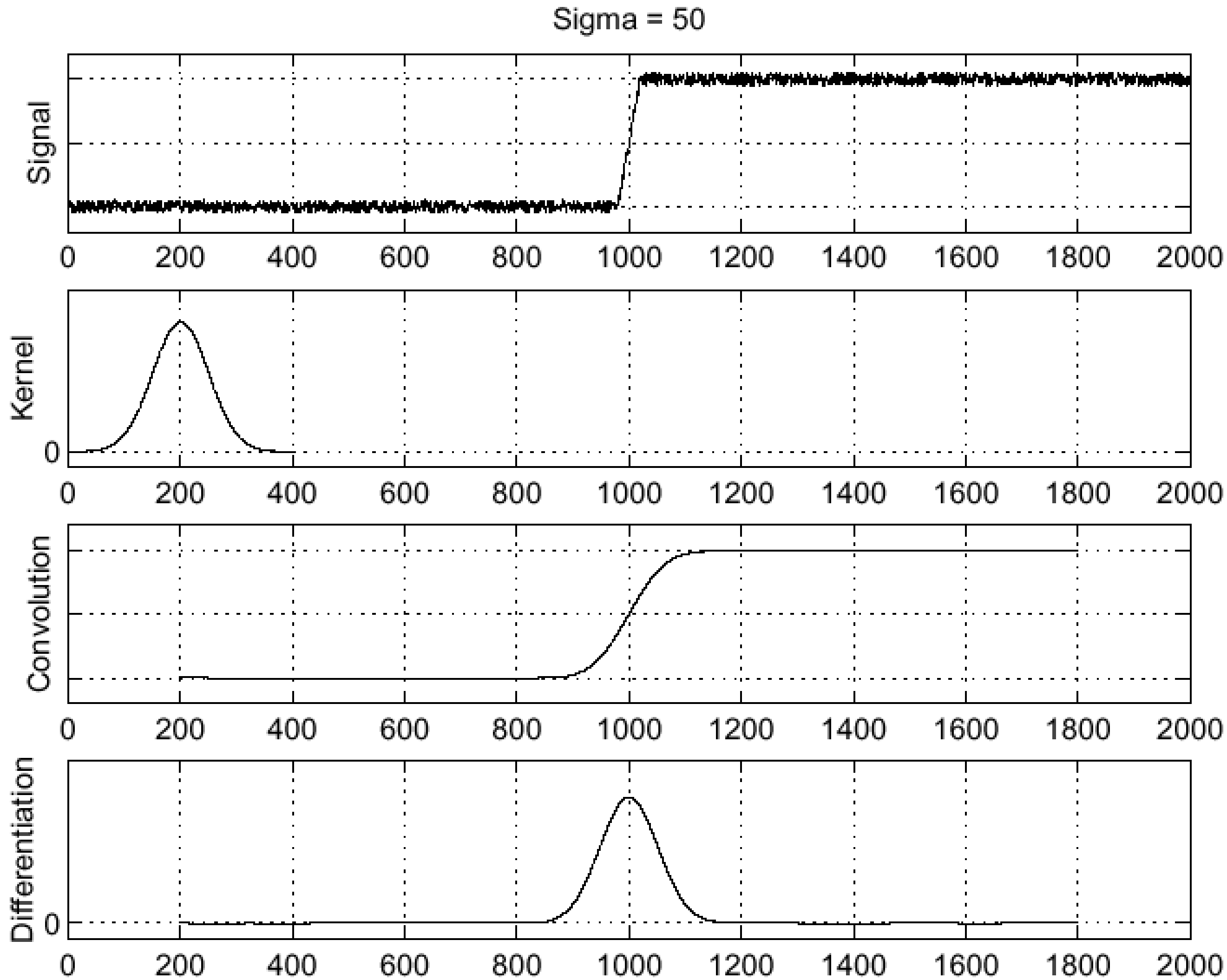
Solution: Smooth First

f

h

$h \star f$

$\frac{\partial}{\partial x}(h \star f)$



Where is the edge? **Look for peaks in** $\frac{\partial}{\partial x}(h \star f)$

Derivative Theorem of Convolution

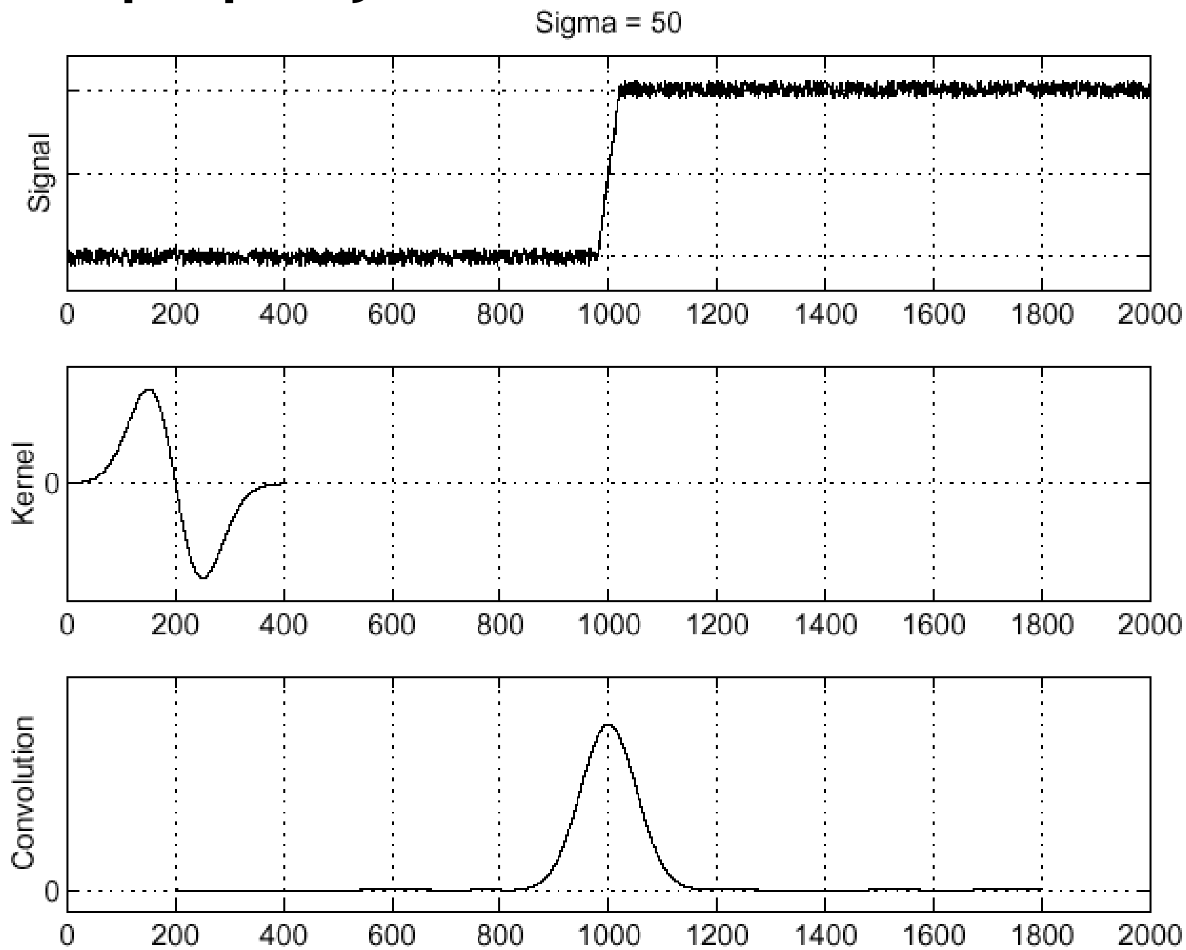
$$\frac{\partial}{\partial x}(h \star f) = (\frac{\partial}{\partial x}h) \star f$$

- **Differentiation property of convolution.**

f

$\frac{\partial}{\partial x}h$

$(\frac{\partial}{\partial x}h) \star f$



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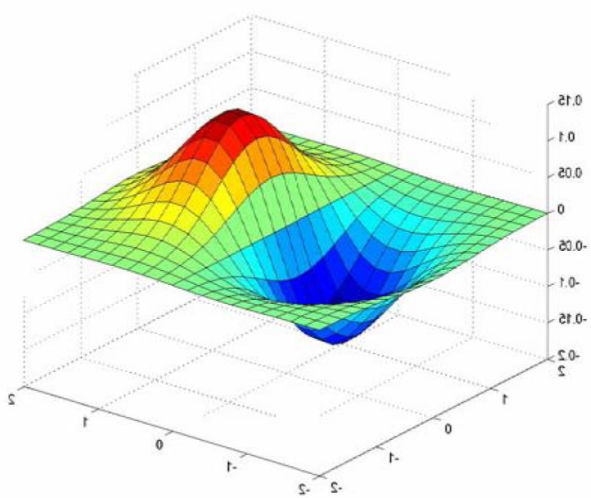
Derivative of Gaussian Filter

$$g * (h * I) = (g * h) * I$$

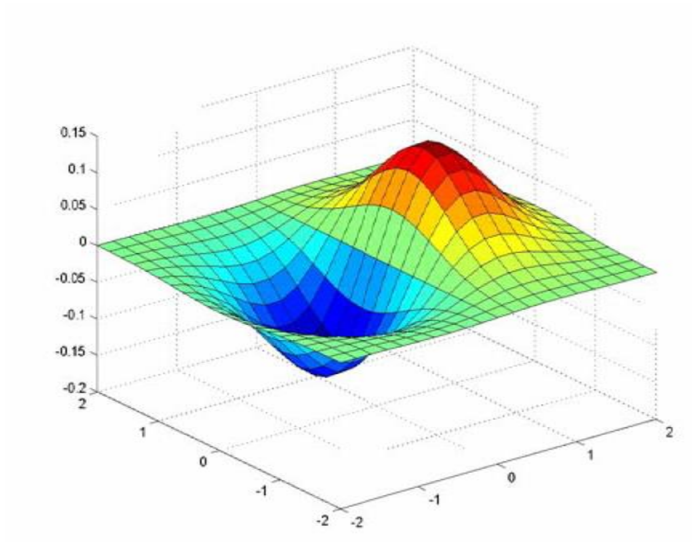
$$\begin{bmatrix} 1 & -1 \end{bmatrix} * \begin{bmatrix} 0.0030 & 0.0133 & 0.0219 & 0.0133 & 0.0030 \\ 0.0133 & 0.0596 & 0.0983 & 0.0596 & 0.0133 \\ 0.0219 & 0.0983 & 0.1621 & 0.0983 & 0.0219 \\ 0.0133 & 0.0596 & 0.0983 & 0.0596 & 0.0133 \\ 0.0030 & 0.0133 & 0.0219 & 0.0133 & 0.0030 \end{bmatrix}$$

Why is this preferable?

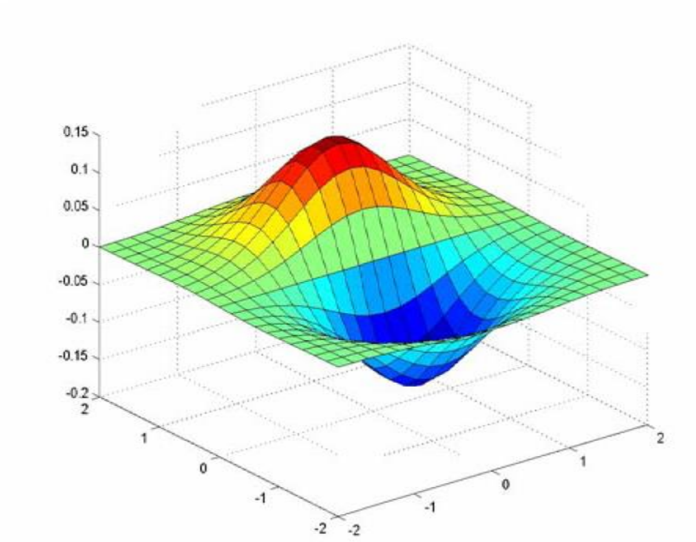


Slide adapted from Kristen Grauman

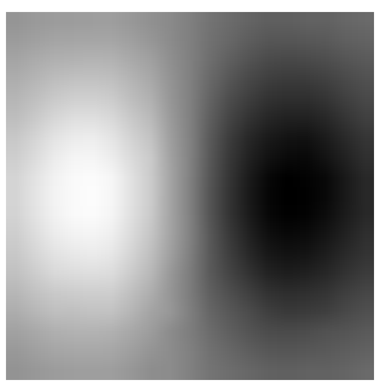
Derivative of Gaussian Filters

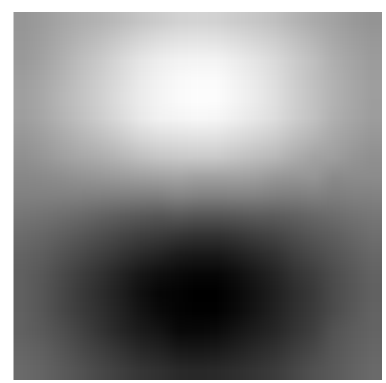


x-direction



y-direction





Source: Svetlana Lazebnik

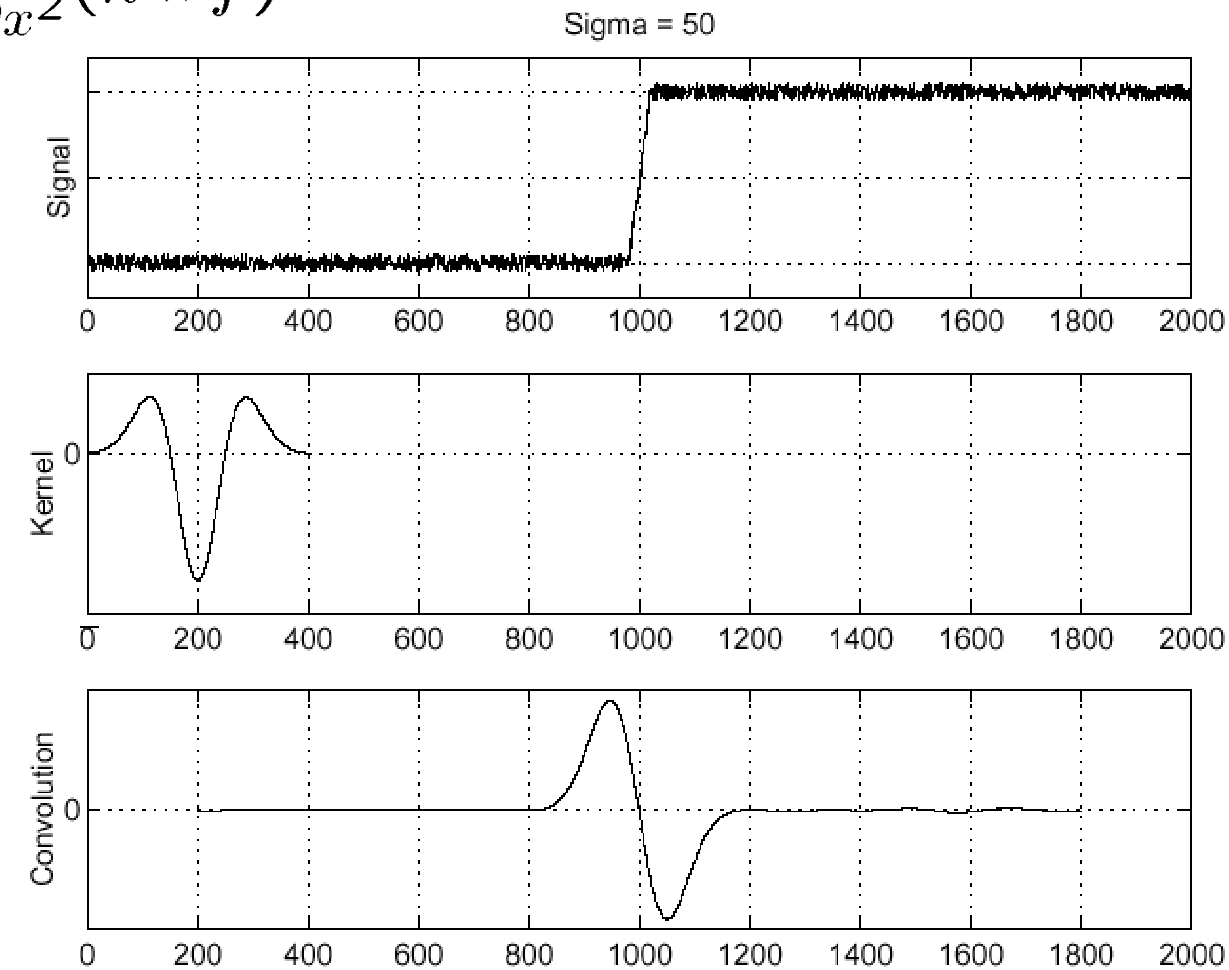
Laplacian of Gaussian (LoG)

- **Consider** $\frac{\partial^2}{\partial x^2}(h \star f)$

f

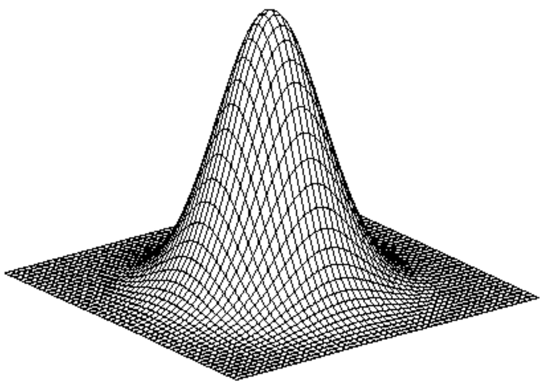
$\frac{\partial^2}{\partial x^2}h$

$(\frac{\partial^2}{\partial x^2}h) \star f$



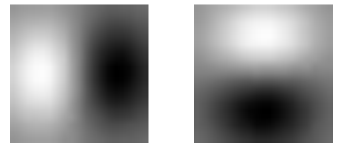
Where is the edge? **Zero-crossings of bottom graph**

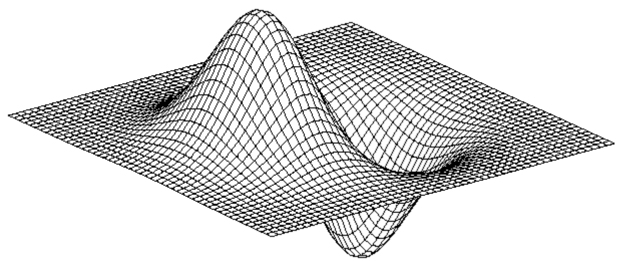
Summary: 2D Edge Detection Filters



Gaussian

$$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$

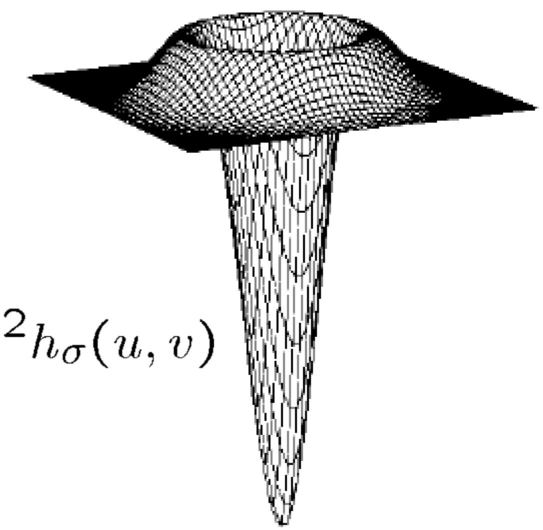




Derivative of Gaussian

$$\frac{\partial}{\partial x} h_\sigma(u,v)$$

Laplacian of Gaussian



$$\nabla^2 h_\sigma(u,v)$$

- ∇^2 is the Laplacian operator:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

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Edge Detection

- Goal: map image from 2D array of pixels to a set of curves or line segments or contours.
- Why?

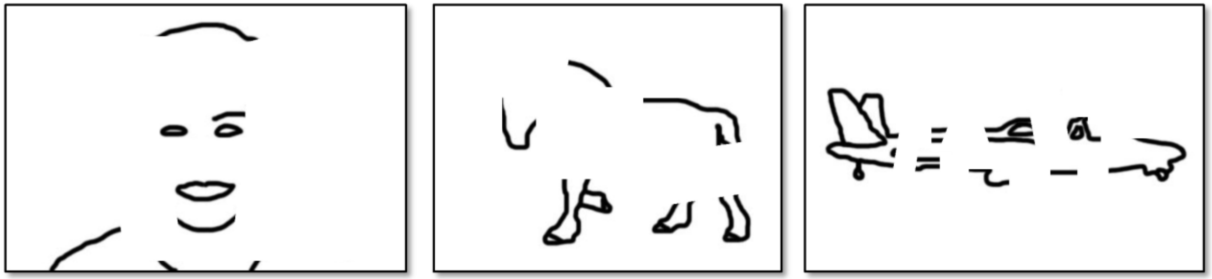
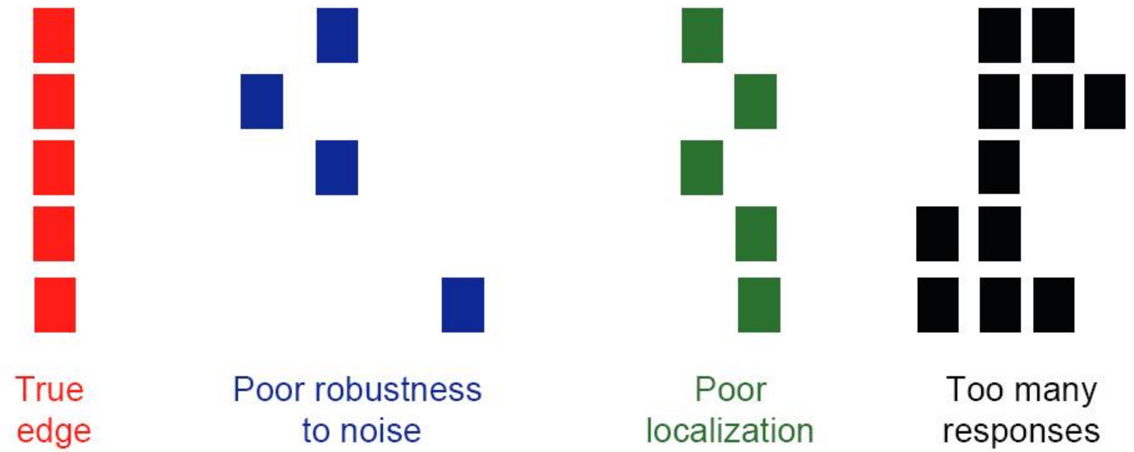


Figure from J. Shotton et al., PAMI 2007

- Main idea: look for strong gradients, post-process

Designing an Edge Detector

- Criteria for an "optimal" edge detector:
  - Good detection: the optimal detector should minimize the probability of false positives (detecting spurious edges caused by noise), as well as that of false negatives (missing real edges).
  - Good localization: the edges detected should be as close as possible to the true edges.
  - Single response: the detector should return one point only for each true edge point; that is, minimize the number of local maxima around the true edge.



Source: Li Fei-Fei

Gradients → Edges

Primary edge detection steps

1. Smoothing: suppress noise
2. Edge enhancement: filter for contrast
3. Edge localization
   - Determine which local maxima from filter output are actually edges vs. noise
   - Thresholding, thinning

- Two issues
  - At what scale do we want to extract structures?
  - How sensitive should the edge extractor be?

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Scale: Effect of σ on Derivatives

σ = 1 pixel σ = 3 pixels

- The apparent structures differ depending on Gaussian's scale parameter.

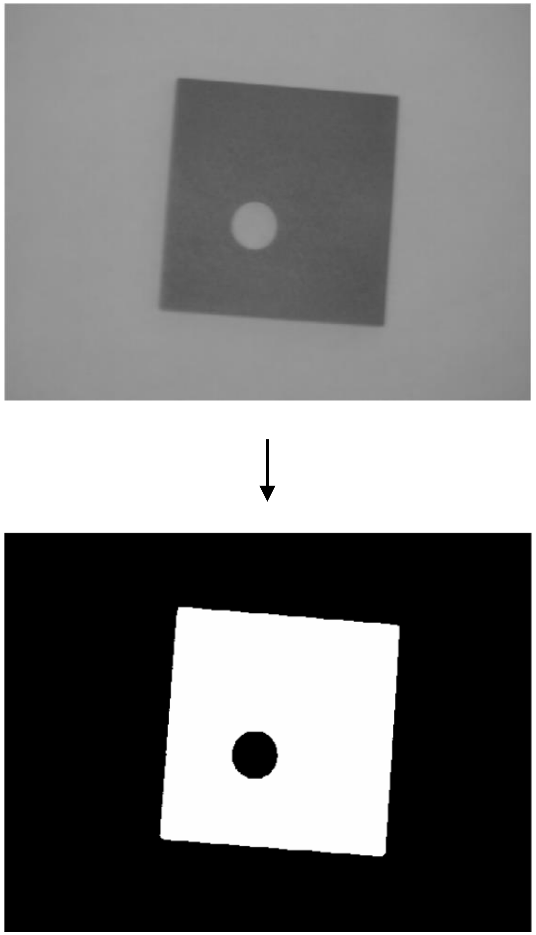
⇒ Larger values: larger-scale edges detected

⇒ Smaller values: finer features detected

Sensitivity: Recall Thresholding

- Choose a threshold t
- Set any pixels less than t to zero (off).
- Set any pixels greater than or equal t to one (on).

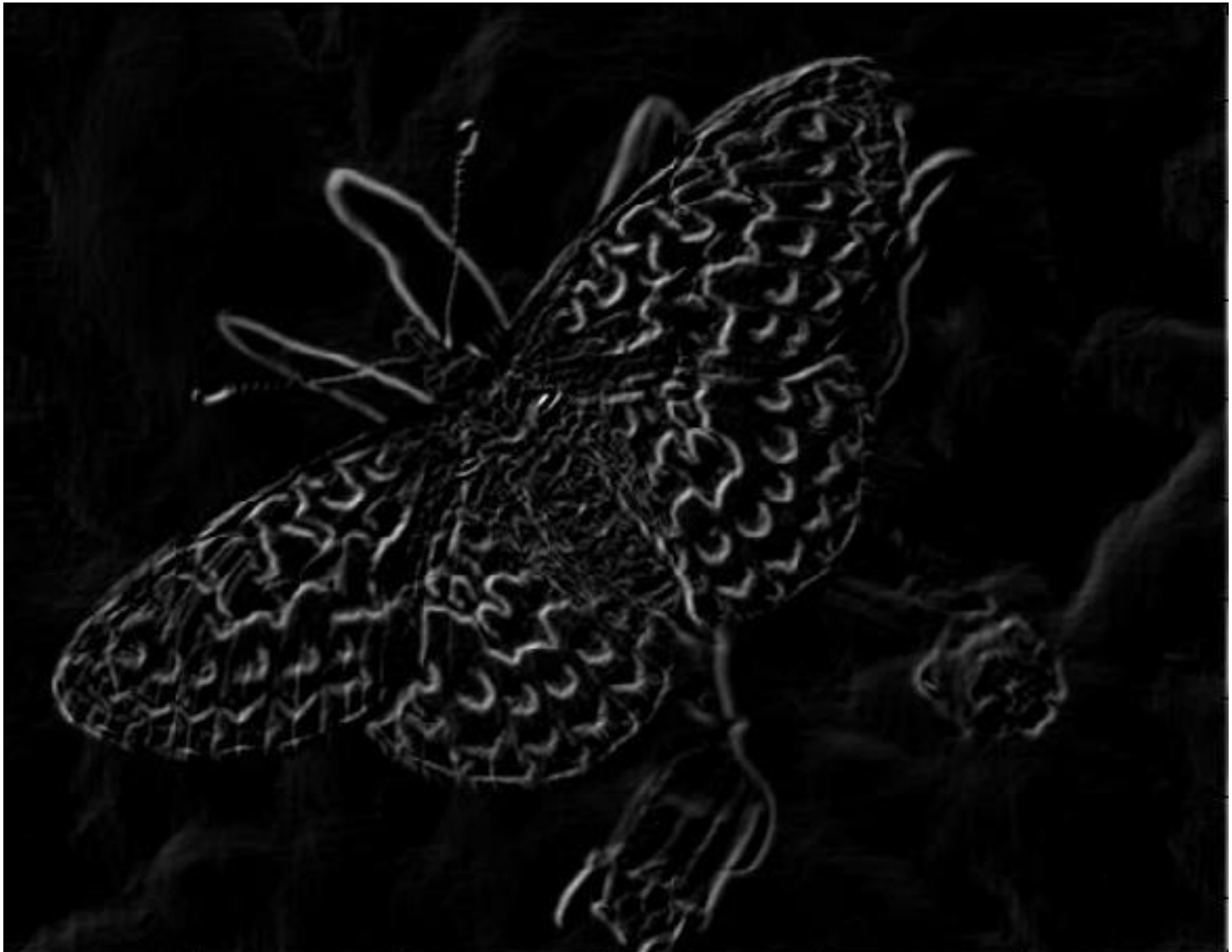
$$F_T[i,j] = \begin{cases} 1, & \text{if } F[i,j] \geq t \\ 0, & \text{otherwise} \end{cases}$$



Original Image

Slide credit: Kristen Grauman

Gradient Magnitude Image



Slide credit: Kristen Grauman

Thresholding with a Lower Threshold

Slide credit: Kristen Grauman

Thresholding with a Higher Threshold

Slide credit: Kristen Grauman

Canny Edge Detector

- Probably the most widely used edge detector in computer vision
- Theoretical model: step-edges corrupted by additive Gaussian noise
- Canny has shown that the first derivative of the Gaussian closely approximates the operator that optimizes the product of *signal-to-noise ratio* and localization.

J. Canny, A Computational Approach To Edge Detection, *IEEE Trans. Pattern Analysis and Machine Intelligence*, 8:679-714, 1986.

Source: Li Fei-Fei

Canny Edge Detector

1. Filter image with derivative of Gaussian
2. Find magnitude and orientation of gradient
3. Non-maximum suppression:
   - Thin multi-pixel wide “ridges” down to single pixel width
4. Linking and thresholding (hysteresis):
   - Define two thresholds: low and high
   - Use the high threshold to start edge curves and the low threshold to continue them

- MATLAB:

```
>> edge(image, 'canny');
>> help edge
```


Source: D. Lowe, L. Fei-Fei

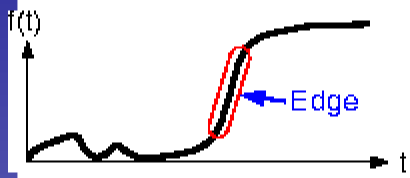
The Canny Edge Detector

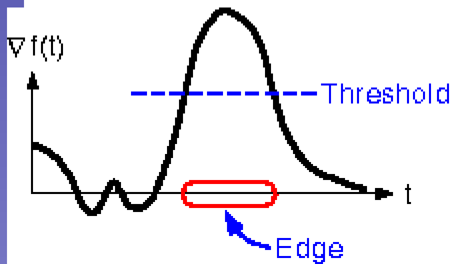
Original image (Lena)

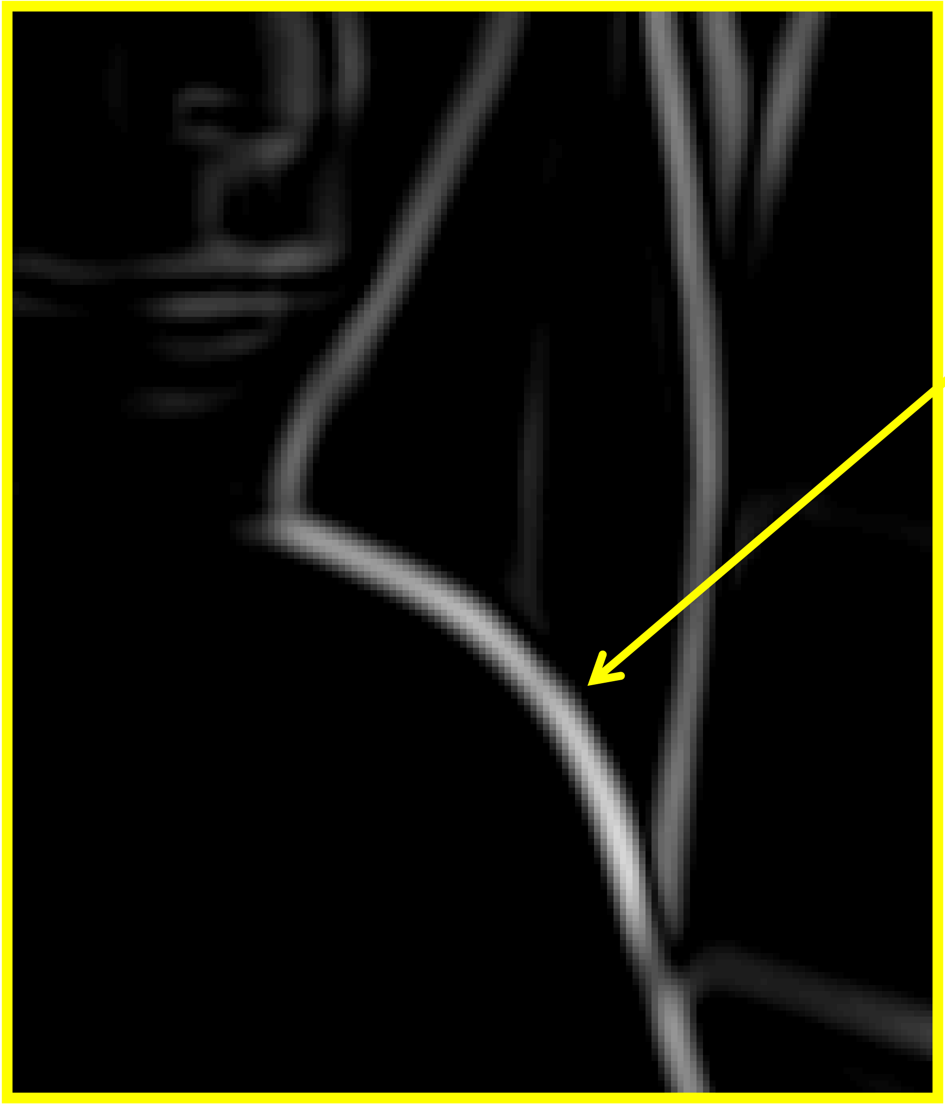
The Canny Edge Detector

Gradient magnitude

The Canny Edge Detector

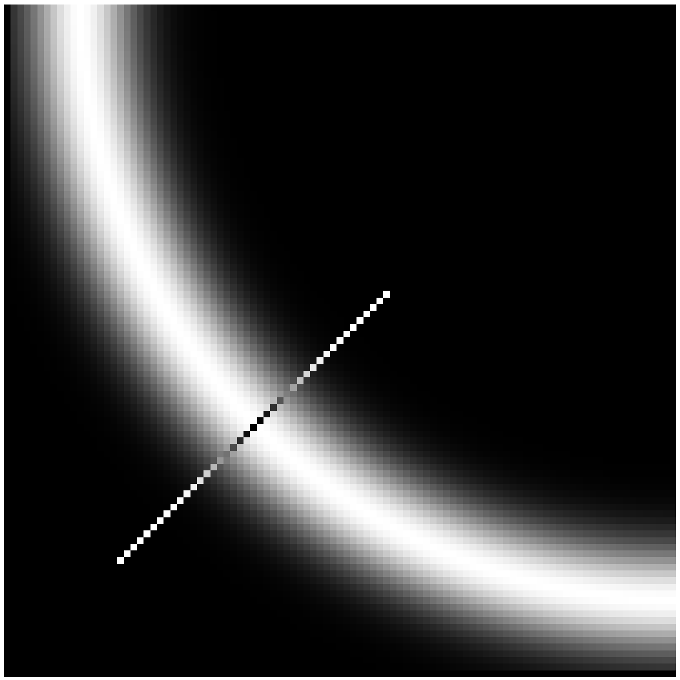


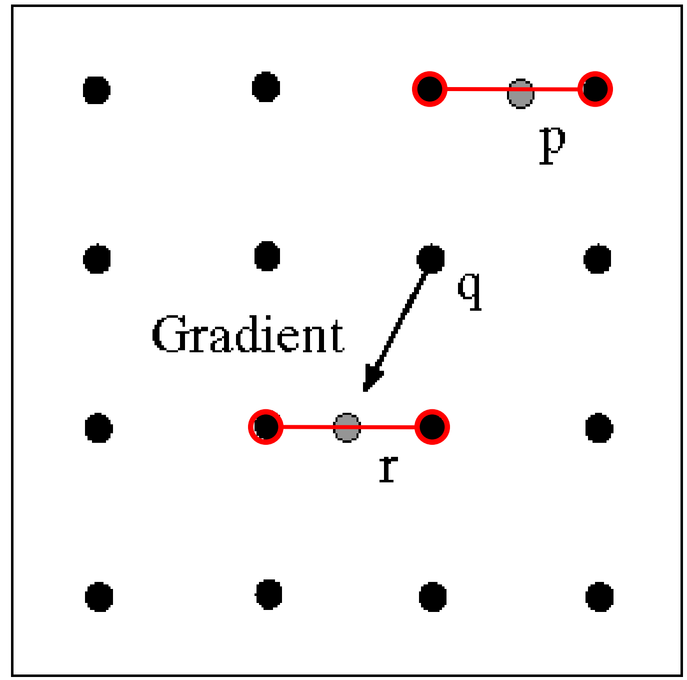




How to turn these thick regions of the gradient into curves?

Non-Maximum Suppression





- **Check if pixel is local maximum along gradient direction, select single max across width of the edge**
  - **Requires checking interpolated pixels p and r**
  - **$\Rightarrow$ Linear interpolation based on gradient direction**

Source: Forsyth & Ponce

The Canny Edge Detector

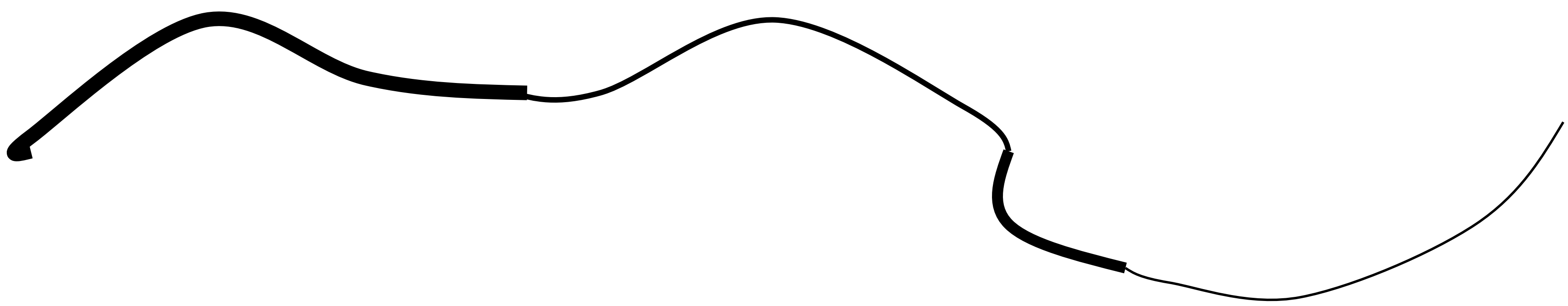
Problem: pixels along this edge didn't survive the thresholding.

Thinning
(non-maximum suppression)

Solution: Hysteresis Thresholding

- Hysteresis: A lag or momentum factor
- Idea: Maintain two thresholds k_{high} and k_{low}
  - Use k_{high} to find strong edges to start edge chain
  - Use k_{low} to find weak edges which continue edge chain
- Typical ratio of thresholds is roughly

$$k_{high} \,/\, k_{low} = 2$$



Source: D. Lowe, S. Seitz

Hysteresis Thresholding

Original image

High threshold (strong edges)

Low threshold (weak edges)

courtesy of G. Loy

Hysteresis threshold

Source: L. Fei-Fei

Object Boundaries vs. Edges

Background

Texture

Shadows

Slide credit: Kristen Grauman

Edge Detection is Just the Beginning…

Image | Human segmentation | Gradient magnitude

- **Berkeley segmentation database:**
  http://www.eecs.berkeley.edu/Research/Projects/CS/vision/grouping/segbench/

Source: L. Lazebnik

References and Further Reading

- **Background information on linear filters and their connection with the Fourier transform can be found in Chapter 7 of F&P. Additional information on edge detection is available in Chapter 8.**
  - D. Forsyth, J. Ponce,
    Computer Vision – A Modern Approach.
    Prentice Hall, 2003