

# Computer Vision - Lecture 8

## Recognition with Global Representations

24.11.2015

Bastian Leibe

RWTH Aachen

<http://www.vision.rwth-aachen.de>

[leibe@vision.rwth-aachen.de](mailto:leibe@vision.rwth-aachen.de)

# Course Outline

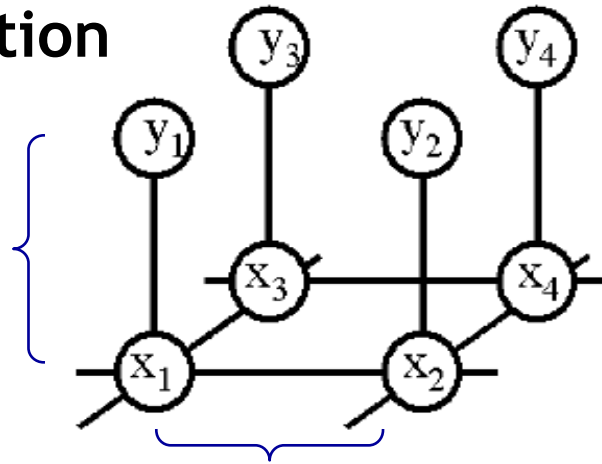
- **Image Processing Basics**
- **Segmentation**
  - Segmentation and Grouping
  - Segmentation as Energy Minimization
- **Recognition & Categorization**
  - **Global Representations**
  - Sliding-Window Object Detection
  - Image Classification
- **Local Features & Matching**
- **3D Reconstruction**
- **Motion and Tracking**

# Recap: MRFs for Image Segmentation

- MRF formulation

Unary potentials

$$\phi(x_i, y_i)$$



Pairwise potentials

$$\psi(x_i, x_j)$$

⇒ Minimize the energy

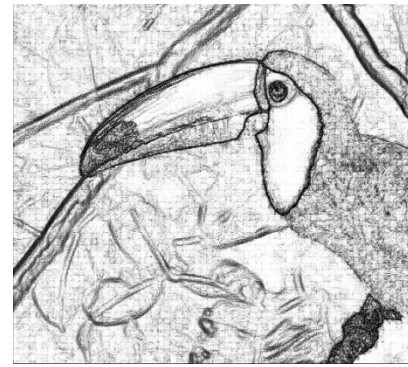
$$E(\mathbf{x}, \mathbf{y}) = \sum_i \phi(x_i, y_i) + \sum_{i,j} \psi(x_i, x_j)$$



Data (D)



Unary likelihood



Pair-wise Terms

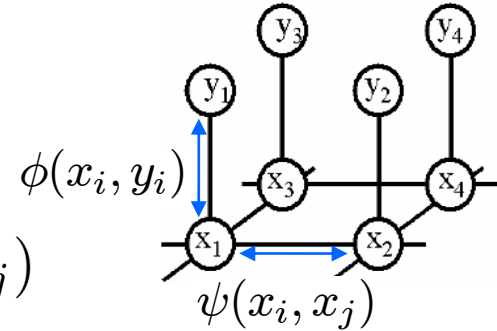


MAP Solution

# Recap: Energy Formulation

- Energy function

$$E(\mathbf{x}, \mathbf{y}) = \sum_i \underbrace{\phi(x_i, y_i)}_{\text{Unary potentials}} + \sum_{i,j} \underbrace{\psi(x_i, x_j)}_{\text{Pairwise potentials}}$$

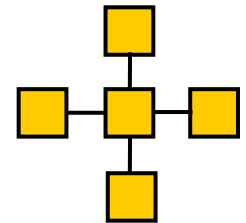


- Unary potentials  $\phi$

- Encode local information about the given pixel/patch
- How likely is a pixel/patch to belong to a certain class (e.g. foreground/background)?

- Pairwise potentials  $\psi$

- Encode neighborhood information
- How different is a pixel/patch's label from that of its neighbor? (e.g. based on intensity/color/texture difference, edges)



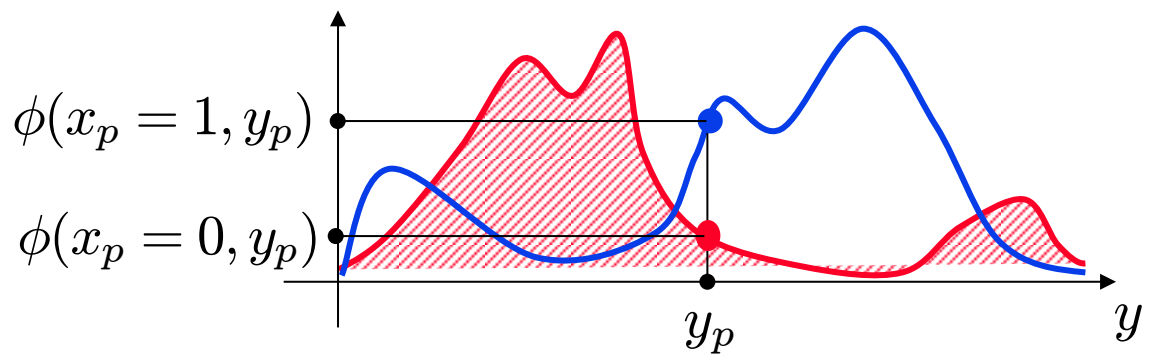
# Recap: How to Set the Potentials?

- Unary potentials

- E.g. color model, modeled with a Mixture of Gaussians

$$\phi(x_i, y_i; \theta_\phi) = \log \sum_k \theta_\phi(x_i, k) p(k|x_i) \mathcal{N}(y_i; \bar{y}_k, \Sigma_k)$$

⇒ Learn color distributions for each label



# Recap: How to Set the Potentials?

- Pairwise potentials

- Potts Model

$$\psi(x_i, x_j; \theta_\psi) = \theta_\psi \delta(x_i \neq x_j)$$

- Simplest discontinuity preserving model.
- Discontinuities between any pair of labels are penalized equally.
- Useful when labels are unordered or number of labels is small.

- Extension: “Contrast sensitive Potts model”

$$\psi(x_i, x_j, g_{ij}(\mathbf{y}); \theta_\psi) = -\theta_\psi g_{ij}(\mathbf{y}) \delta(x_i \neq x_j)$$

where

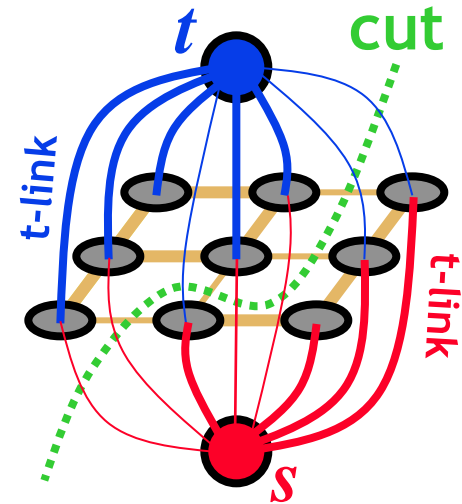
$$g_{ij}(\mathbf{y}) = e^{-\beta \|y_i - y_j\|^2} \quad \beta = \frac{1}{2} \left( \text{avg} (\|y_i - y_j\|^2) \right)^{-1}$$

⇒ Discourages label changes except in places where there is also a large change in the observations.

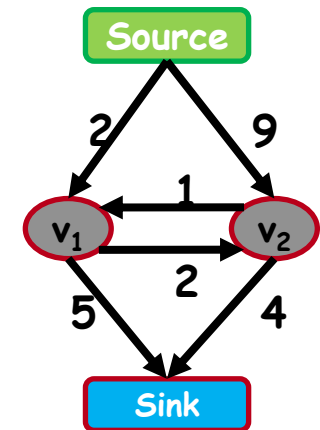
# Recap: Graph-Cuts Energy Minimization

- Solve an equivalent graph cut problem
  1. Introduce extra nodes: source and sink
  2. Weight connections to source/sink (t-links) by  $\phi(x_i = s)$  and  $\phi(x_i = t)$ , respectively.
  3. Weight connections between nodes (n-links) by  $\psi(x_i, x_j)$ .
  4. Find the minimum cost cut that separates source from sink.

⇒ Solution is equivalent to minimum of the energy.



- s-t Mincut can be solved efficiently
  - Dual to the well-known max flow problem
  - Very efficient algorithms available for regular grid graphs (1-2 MPixels/s)
  - Globally optimal result for 2-class problems



# Recap: When Can s-t Graph Cuts Be Applied?

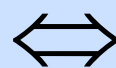
$$E(L) = \sum_p E_p(L_p) + \sum_{pq \in N} E(L_p, L_q)$$

Unary potentials                      Pairwise potentials

t-links                                      n-links                       $L_p \in \{s, t\}$

- s-t graph cuts can only globally minimize **binary energies** that are **submodular**. [Boros & Hummer, 2002, Kolmogorov & Zabih, 2004]

$E(L)$  can be minimized  
by s-t graph cuts



$$E(s, s) + E(t, t) \leq E(s, t) + E(t, s)$$

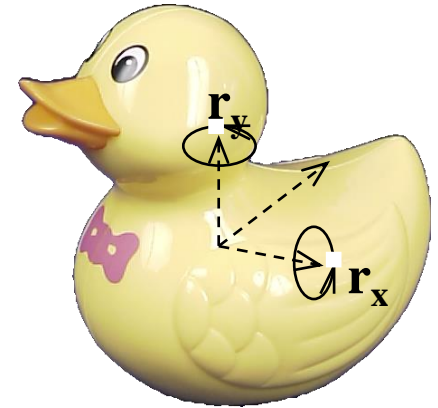
Submodularity (“convexity”)

- Submodularity is the discrete equivalent to convexity.
  - Implies that every local energy minimum is a global minimum.
  - ⇒ Solution will be globally optimal.

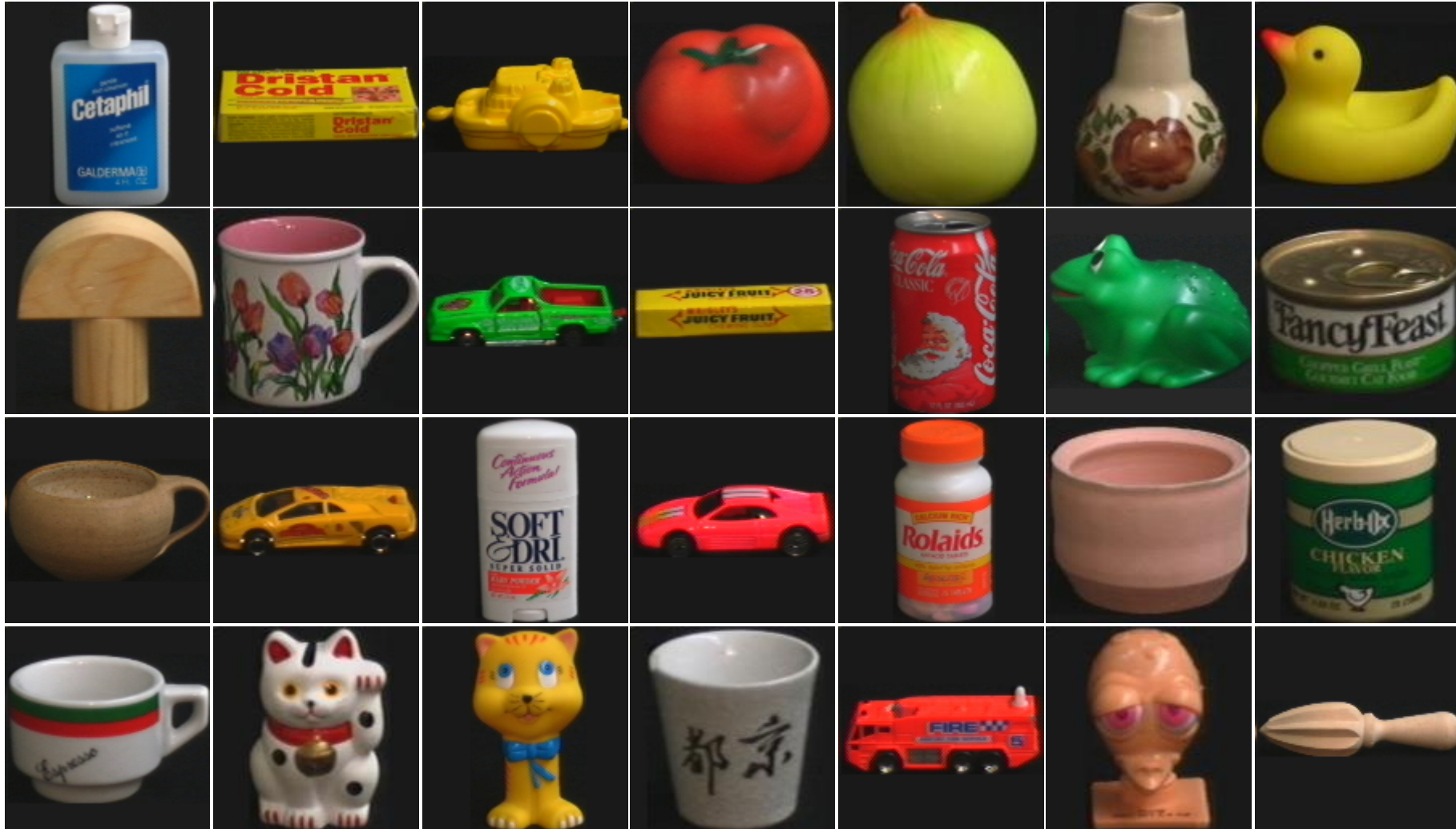


# Topics of This Lecture

- **Object Recognition**
  - Appearance-based recognition
  - Global representations
  - Color histograms
- **Recognition using histograms**
  - Histogram comparison measures
  - Histogram backprojection
  - Multidimensional histograms
  - Extension: colored derivatives

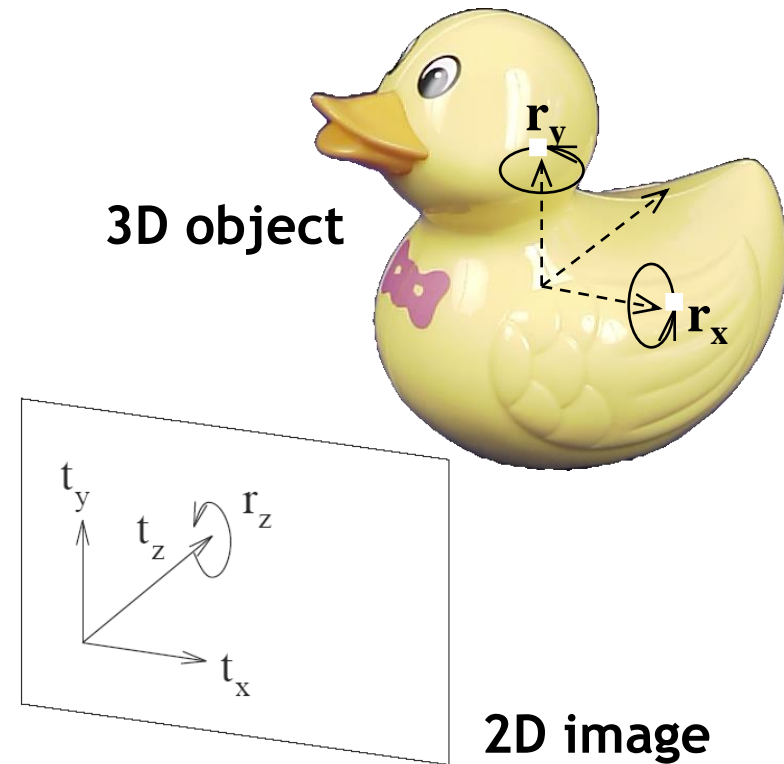


# Object Recognition



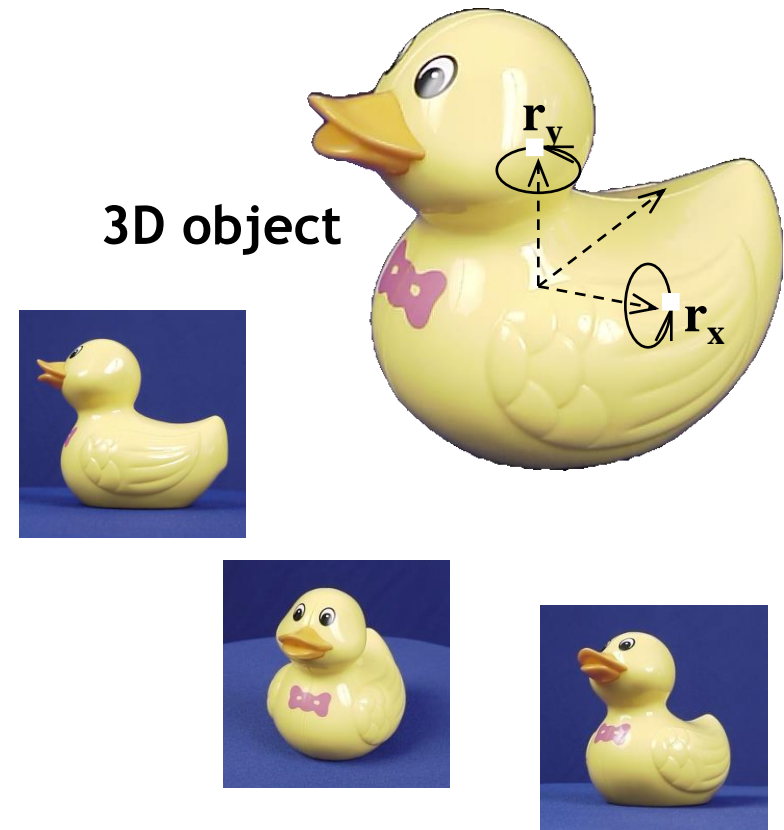
# Challenges

- Viewpoint changes
  - Translation
  - Image-plane rotation
  - Scale changes
  - Out-of-plane rotation
- Illumination
- Noise
- Clutter
- Occlusion



# Appearance-Based Recognition

- Basic assumption
  - Objects can be represented by a set of images (“appearances”).
  - For recognition, it is sufficient to just compare the 2D appearances.
  - No 3D model is needed.

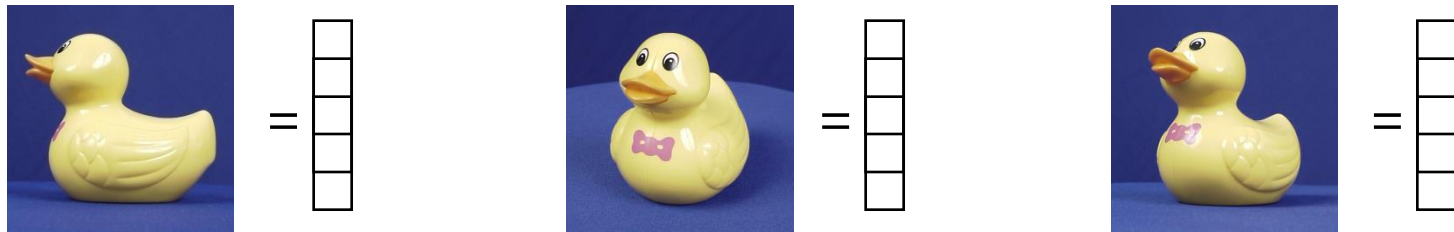


⇒ Fundamental paradigm shift in the 90's

# Global Representation

- Idea

- Represent each object (view) by a global descriptor.



- For recognizing objects, just match the descriptors.
- Some modes of variation are built into the descriptor, the others have to be incorporated in the training data.
  - E.g., a descriptor can be made invariant to image-plane rotations.
  - Other variations:

### Viewpoint changes

- Translation
- Scale changes
- Out-of-plane rotation

### Illumination

- Noise
- Clutter
- Occlusion

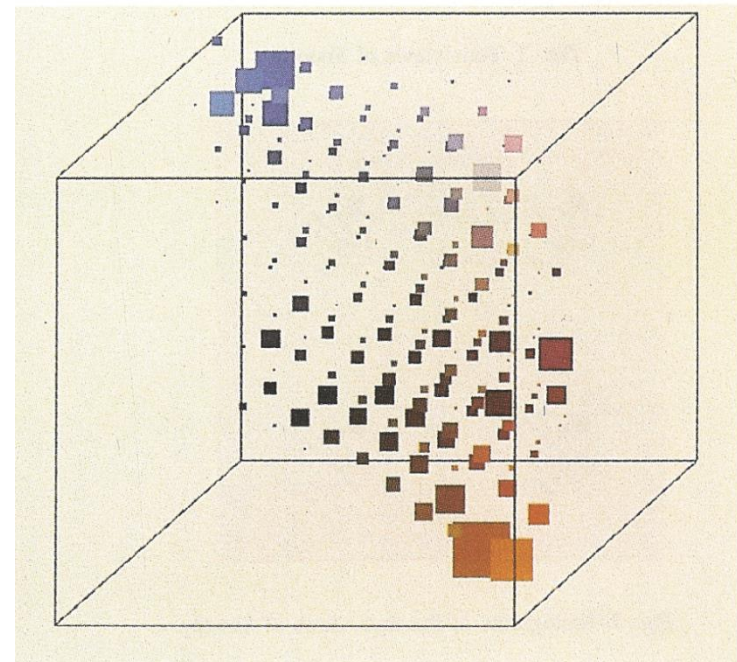
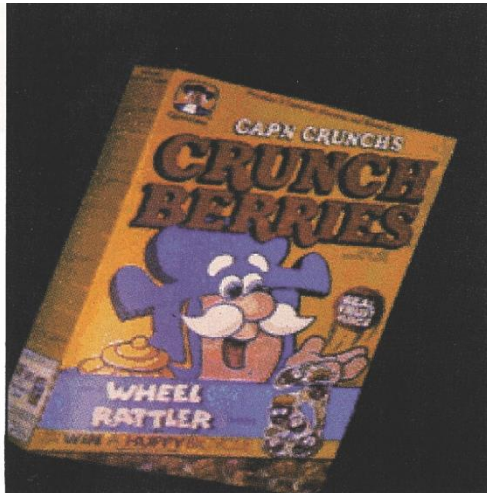
# Color: Use for Recognition

- **Color:**
  - Color stays constant under geometric transformations
  - Local feature
    - Color is defined for each pixel
    - Robust to partial occlusion
- **Idea**
  - Directly use object colors for recognition
  - Better: use **statistics** of object colors



# Color Histograms

- Color statistics
  - Here: RGB as an example
  - Given: tristimulus R,G,B for each pixel
  - Compute 3D histogram
    - $H(R,G,B) = \#(\text{pixels with color } (R,G,B))$



# Color Normalization

- One component of the 3D color space is intensity
  - If a color vector is multiplied by a scalar, the intensity changes, but not the color itself.
  - This means colors can be normalized by the intensity.
    - Intensity is given by  $I = R + G + B$ :
  - „Chromatic representation“

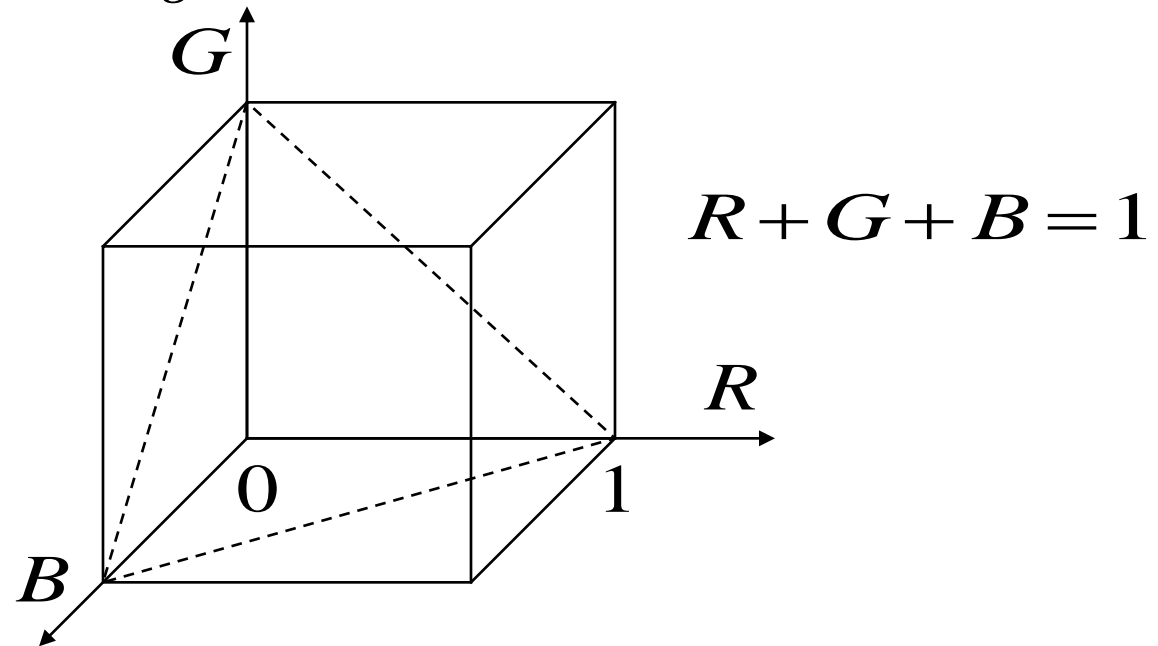
$$r = \frac{R}{R + G + B} \qquad g = \frac{G}{R + G + B}$$

$$b = \frac{B}{R + G + B}$$



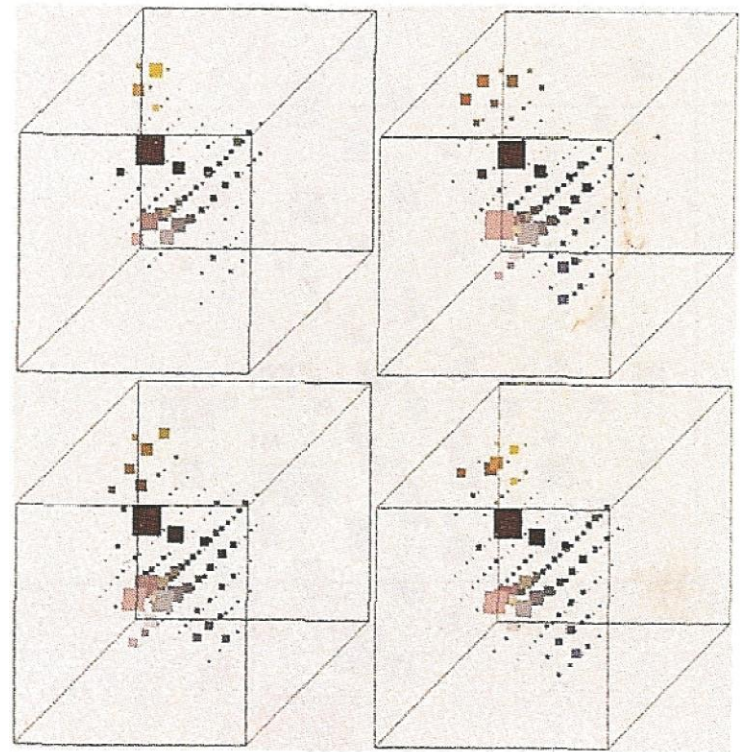
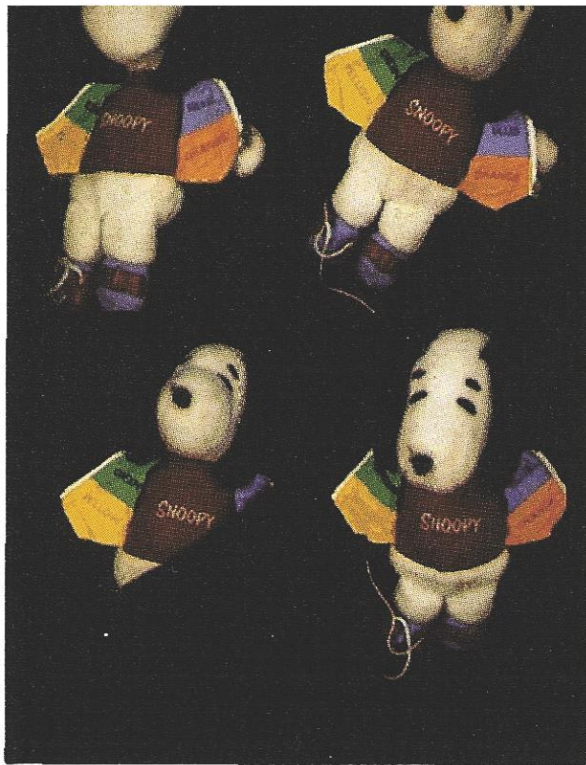
# Color Normalization

- Observation:
  - Since  $r + g + b = 1$ , only 2 parameters are necessary
  - E.g. one can use  $r$  and  $g$
  - and obtains  $b = 1 - r - g$



# Color Histograms

- Robust representation



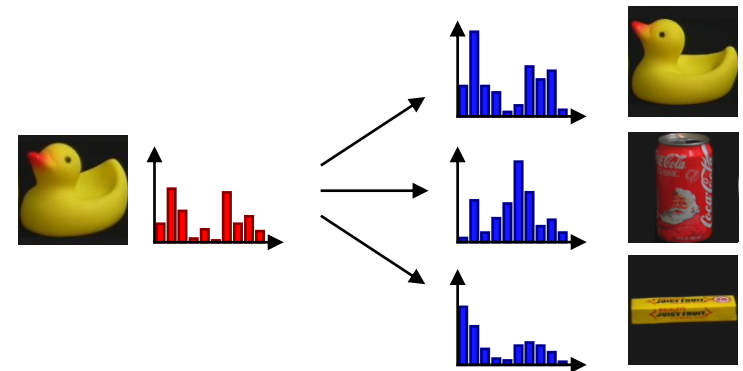
# Color Histograms

- Use for recognition
  - Works surprisingly well
  - In the first paper (1991), 66 objects could be recognized almost without errors



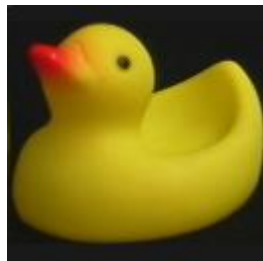
# Topics of This Lecture

- Object Recognition
  - Appearance-based recognition
  - Global representations
  - Color histograms
- Recognition using histograms
  - Histogram comparison measures
  - Histogram backprojection
  - Multidimensional histograms
  - Extension: colored derivatives

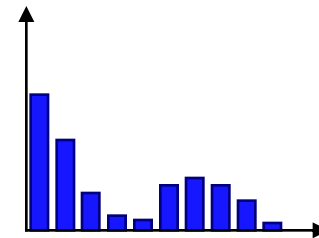
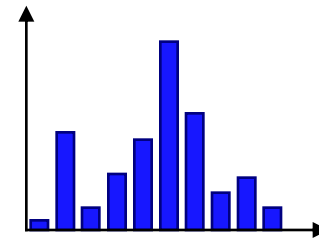
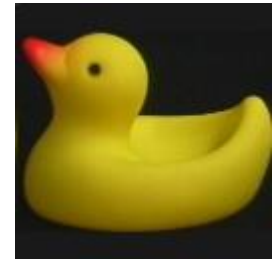
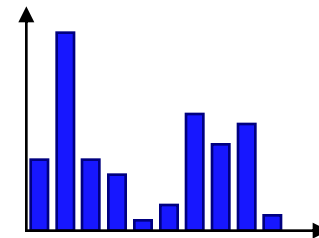
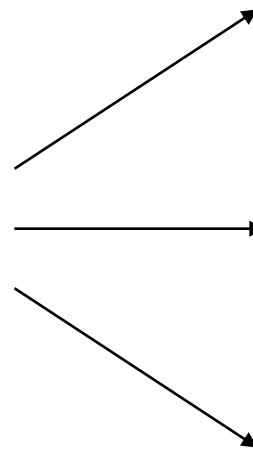
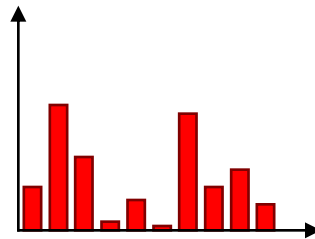


# Recognition Using Histograms

- Histogram comparison



Test image



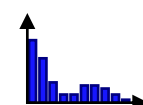
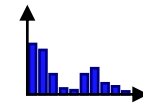
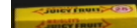
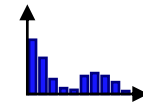
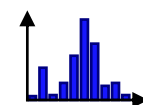
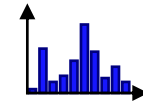
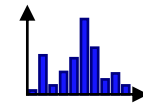
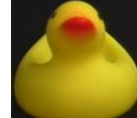
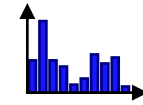
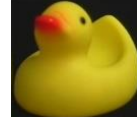
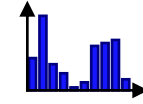
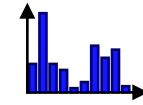
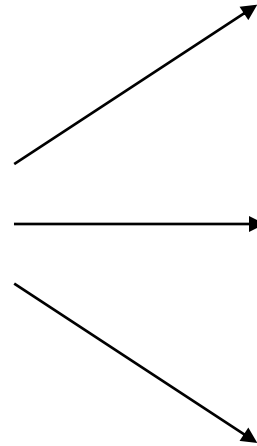
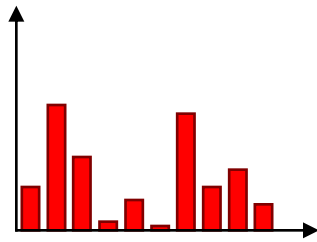
Known objects

# Recognition Using Histograms

- With multiple training views

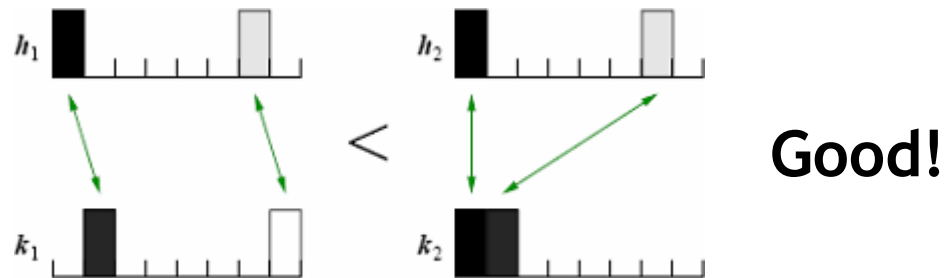
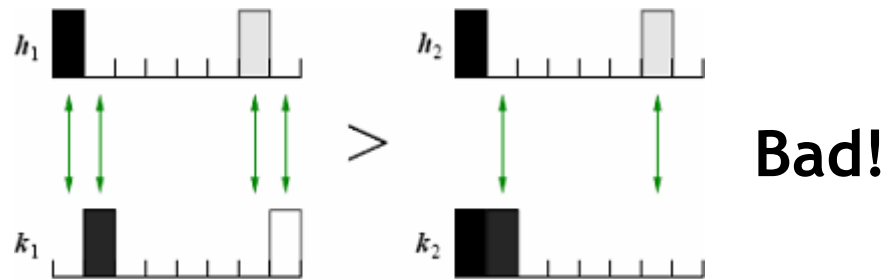


Test image



# What Is a Good Comparison Measure?

- How to define matching cost?

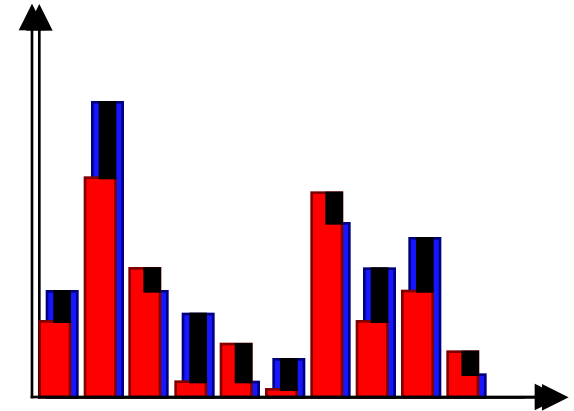


# Comparison Measures: Euclidean Distance

- **Definition**

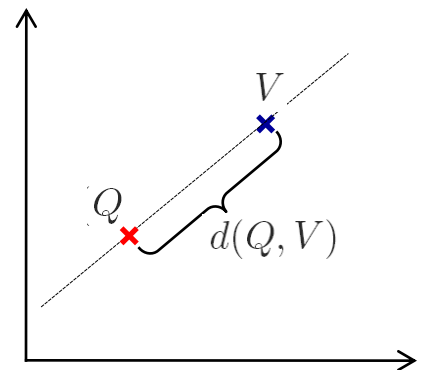
- Euclidean Distance (=L<sub>2</sub> norm)

$$d(Q, V) = \sum_i (q_i - v_i)^2$$



- **Motivation**

- Focuses on the differences between the histograms.
- Interpretation: distance in feature space.
- Range: [0,∞]
- All cells are weighted equally.
- Not very robust to outliers!





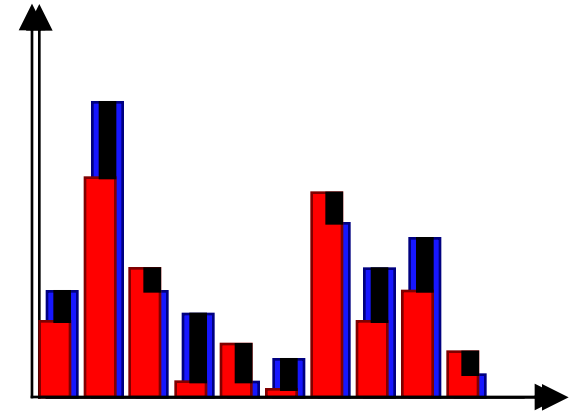
# Comparison Measures: Mahalanobis Distance

## • Definition

- Mahalanobis distance (Quadratic Form)

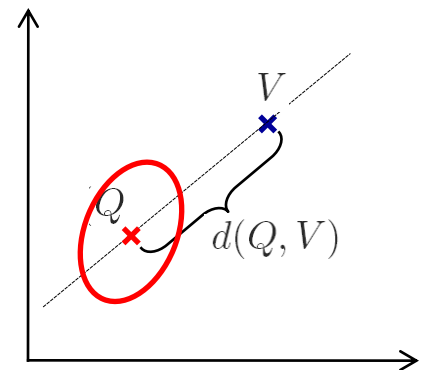
$$d(Q, V) = (Q - V)^T \Sigma^{-1} (Q - V)$$

$$= \sum_i \sum_j \frac{(q_i - v_i)(q_j - v_j)}{\sigma_{ij}}$$



## • Motivation

- Interpretation:
  - Weighted distance in feature space.
  - Compensate for correlated data.
- Range:  $[0, \infty]$
- More robust to certain outliers.

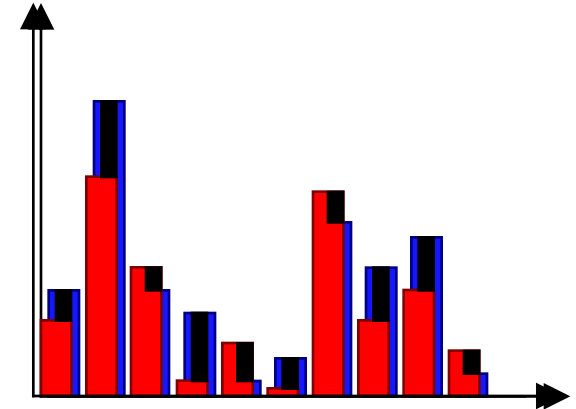


# Comparison Measures: Chi-Square

- **Definition**

- Chi-square

$$\chi^2(Q, V) = \sum_i \frac{(q_i - v_i)^2}{q_i + v_i}$$



- **Motivation**

- **Statistical background:**
  - Test if two distributions are different
  - Possible to compute a significance score
- **Range:  $[0, \infty]$**
- **Cells are not weighted equally!**
- **More robust to outliers than Euclidean distance.**
  - If the histograms contain enough observations...

# Comp. Measures: Bhattacharyya Distance

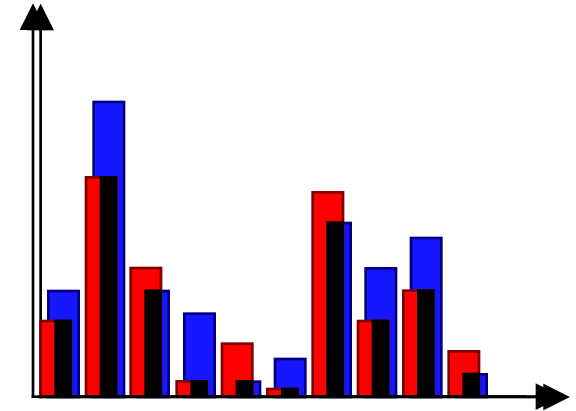
- **Definition**

- Bhattacharyya coefficient

$$BC(Q, V) = \sum_i \sqrt{q_i v_i}$$

- Common distance measure:

$$d_{BC}(Q, V) = \sqrt{1 - BC(Q, V)}$$



- **Motivation**

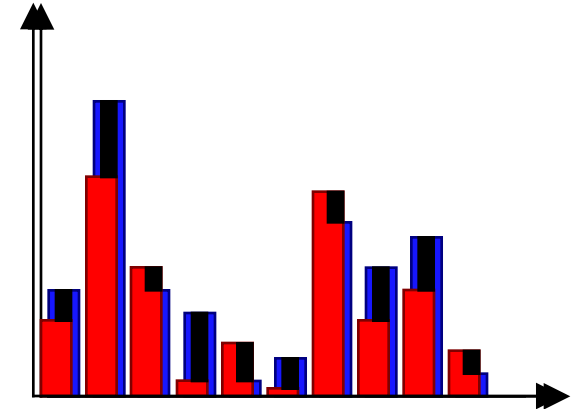
- Statistical background
  - $BC$  measures the statistical separability between two distributions.
- Range:  $[0, \infty]$
- (Reason for  $d_{BC}$ : triangle inequality)

# Comparison Measures: Kullback-Leibler

- Definition

- KL-divergence

$$KL(Q, V) = \sum_i q_i \log \frac{q_i}{v_i}$$



- Motivation

- Information-theoretic background:
  - Measures the expected difference (#bits) required to code samples from distribution  $Q$  when using a code based on  $Q$  vs. based on  $V$ .
  - Also called: *information gain*, *relative entropy*
- Not symmetric!
- Symmetric version: *Jeffreys divergence*

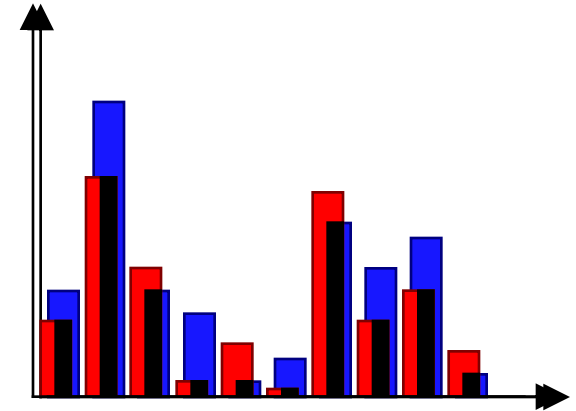
$$JD(Q, V) = KL(Q, V) + KL(V, Q)$$

# Comp. Measures: Histogram Intersection

- **Definition**

- Intersection

$$\cap(Q, V) = \sum_i \min(q_i, v_i)$$



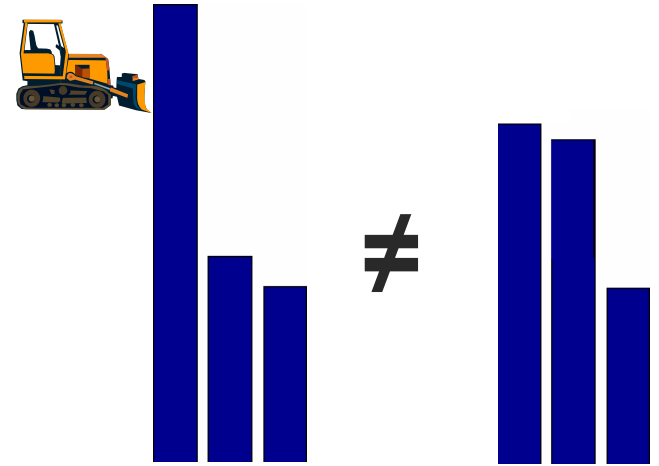
- **Motivation**

- Measures the common part of both histograms
- Range: [0,1]
- For unnormalized histograms, use the following formula

$$\cap(Q, V) = \frac{1}{2} \left( \frac{\sum_i \min(q_i, v_i)}{\sum_i q_i} + \frac{\sum_i \min(q_i, v_i)}{\sum_i v_i} \right)$$

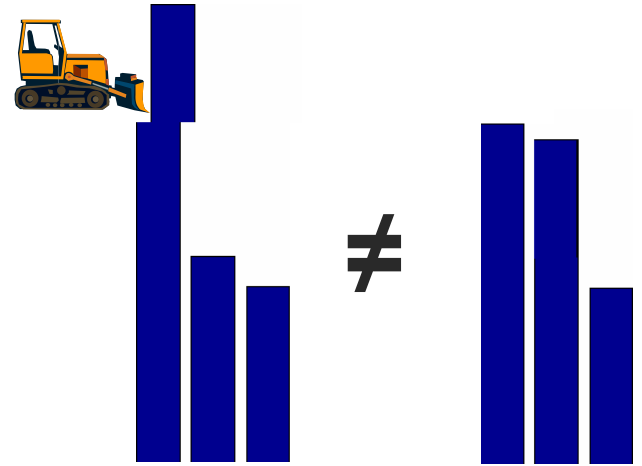
# Comp. Measures: Earth Movers Distance

- Motivation: Moving Earth



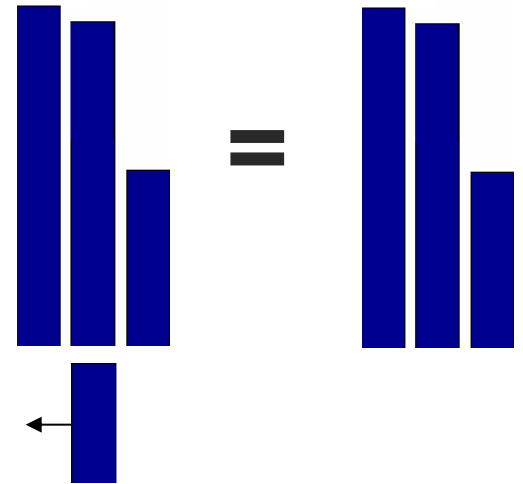
# Comp. Measures: Earth Movers Distance

- Motivation: Moving Earth



# Comp. Measures: Earth Movers Distance

- Motivation: Moving Earth

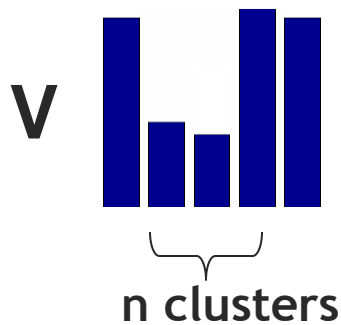
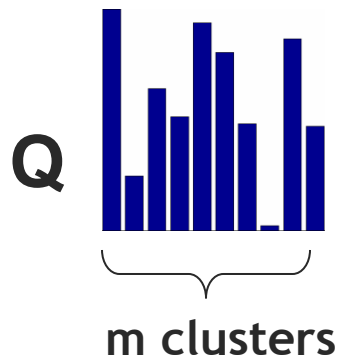


(distance moved) \* (amount moved)



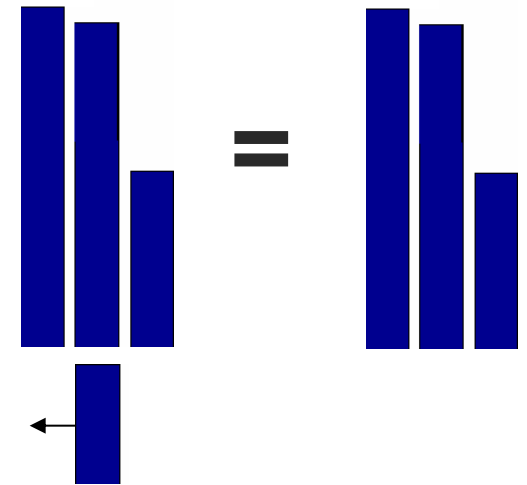
# Comp. Measures: Earth Movers Distance

- Motivation: Moving Earth
  - Linear Programming Problem



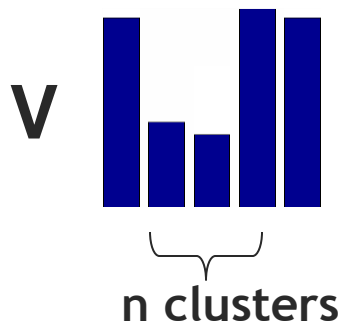
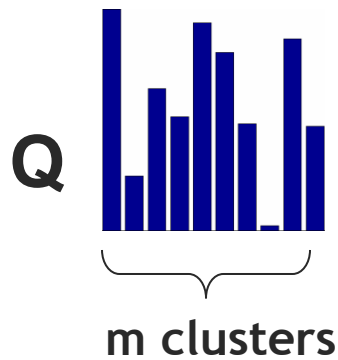
All movements

(distance moved) \* (amount moved)



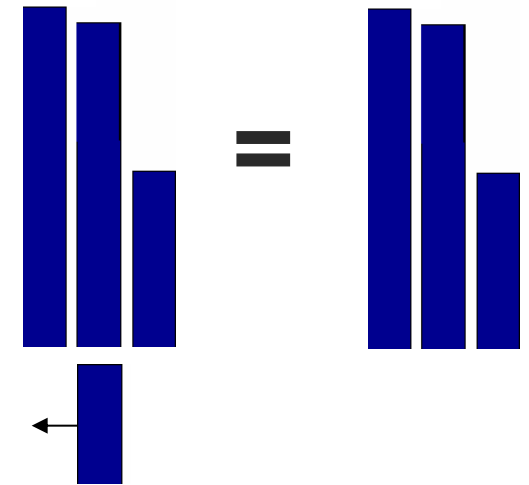
# Comp. Measures: Earth Movers Distance

- Motivation: Moving Earth
  - Linear Programming Problem



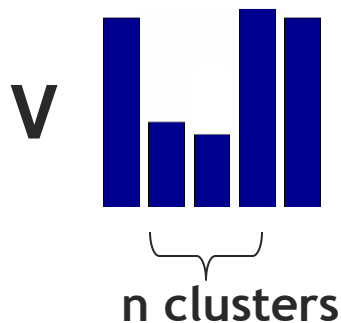
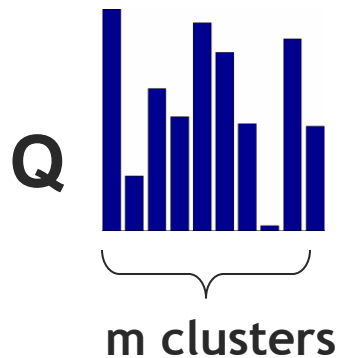
$$\sum_{i=1}^m \sum_{j=1}^n d_{ij} * (\text{amount moved})$$

All movements



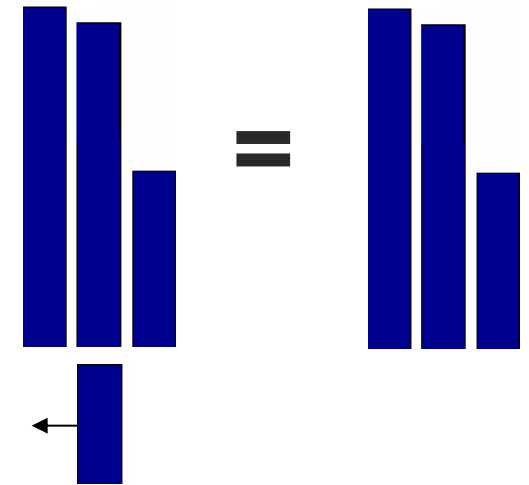
# Comp. Measures: Earth Movers Distance

- Motivation: Moving Earth
  - Linear Programming Problem



$$\sum_{i=1}^m \sum_{j=1}^n d_{ij} f_{ij} = \text{WORK}$$

All movements

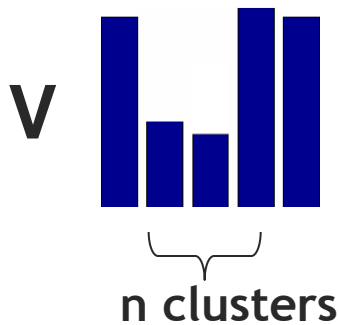
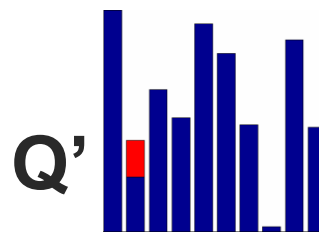
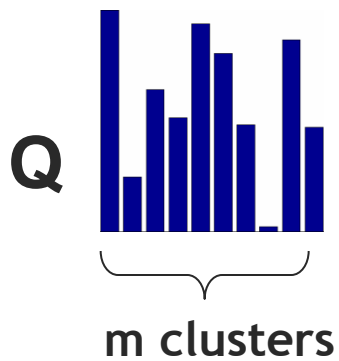


⇒ *What is the minimum amount of work to convert Q into V?*

# EMD Computation

- Constraints

1. Move “earth” only from Q to V

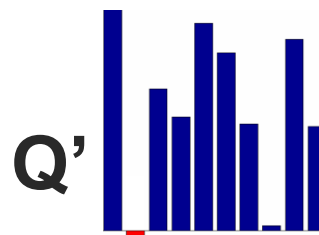
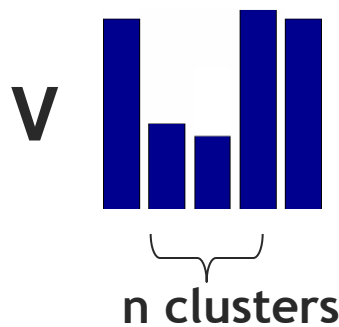
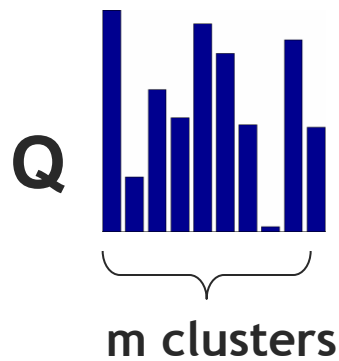


$$f_{ij} \geq 0$$

# EMD Computation

- Constraints

2. Cannot send more “earth” than there is

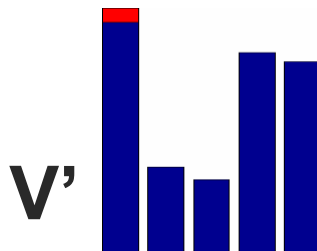
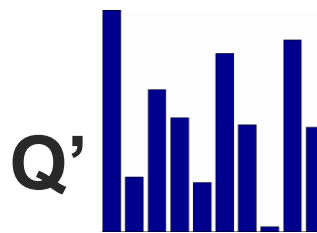
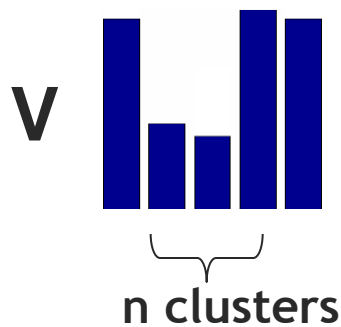
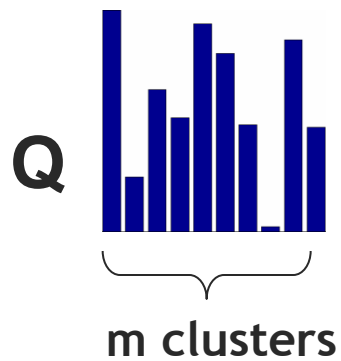


$$\sum_{j=1}^n f_{ij} \leq w_{q_i}$$

# EMD Computation

- Constraints

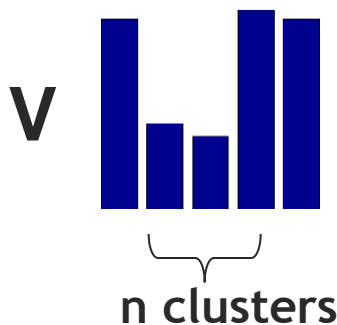
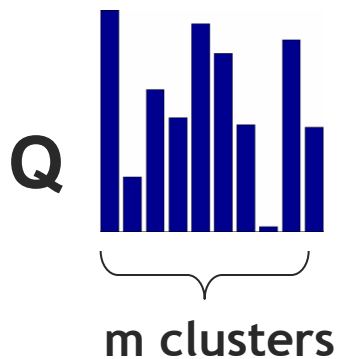
3. V cannot receive more than it can hold



$$\sum_{i=1}^m f_{ij} \leq w_{v_j}$$

# EMD Computation

- Constraints



4. As much “earth” as possible must be moved.

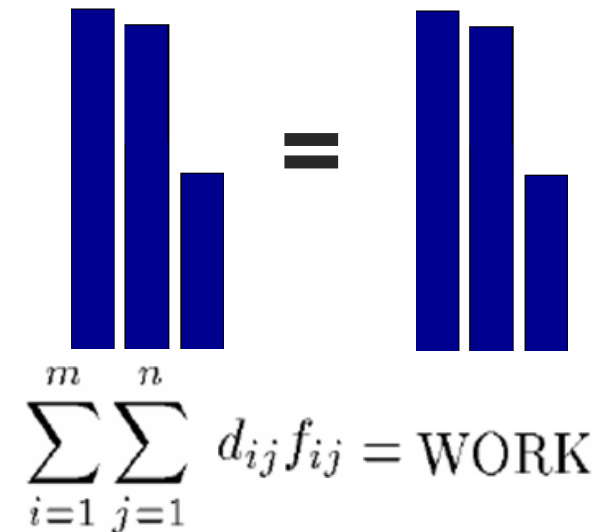
- Either Q must be completely spent or V must be completely filled.

$$\sum_{i=1}^m \sum_{j=1}^n f_{ij} = \min \left( \sum_{i=1}^m w_{q_i}, \sum_{j=1}^n w_{v_j} \right)$$

# Comp. Measures: Earth Movers Distance

- **Motivation: Moving Earth**
  - Linear Programming Problem
  - Distance measure

$$D_{EMD}(Q, V) = \frac{\sum_{i,j} d_{ij} f_{ij}}{\sum_{i,j} f_{ij}}$$

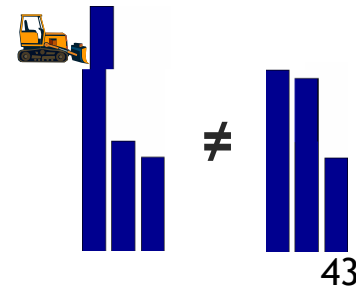
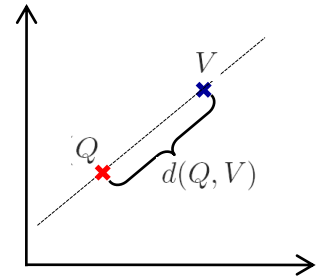


- **Advantages**
  - Nearness measure without quantization
  - Partial matching
  - A true metric
- **Disadvantage: expensive computation**
  - Efficient algorithms available for 1D
  - Approximations for higher dimensions...



# Summary: Comparison Measures

- **Vector space interpretation**
  - Euclidean distance
  - Mahalanobis distance
- **Statistical motivation**
  - Chi-square
  - Bhattacharyya
- **Information-theoretic motivation**
  - Kullback-Leibler divergence, Jeffreys divergence
- **Histogram motivation**
  - Histogram intersection
- **Ground distance**
  - Earth Movers Distance (EMD)



# Comparison for Image Retrieval

Query							
1) 0.00 29020.jpg	2) 0.53 29077.jpg	3) 0.61 157090.jpg	4) 0.61 9045.jpg	5) 0.63 197037.jpg	6) 0.67 20003.jpg	7) 0.70 81005.jpg	8) 0.70 160053.jpg

L2 distance

Query							
1) 0.00 29020.jpg	2) 0.26 29077.jpg	3) 0.43 29017.jpg	4) 0.61 29005.jpg	5) 0.72 197037.jpg	6) 0.73 77047.jpg	7) 0.75 197097.jpg	8) 0.77 20003.jpg

Jeffrey divergence

Query							
1) 0.00 29020.jpg	2) 0.11 29077.jpg	3) 0.19 157090.jpg	4) 0.21 197037.jpg	5) 0.21 81005.jpg	6) 0.21 29017.jpg	7) 0.22 197058.jpg	8) 0.22 77045.jpg

$\chi^2$  statistics

Query							
1) 0.00 29020.jpg	2) 8.16 29077.jpg	3) 12.23 29005.jpg	4) 12.64 29017.jpg	5) 13.82 20003.jpg	6) 14.52 53062.jpg	7) 14.70 29018.jpg	8) 14.78 29019.jpg

Earth Movers Distance

# Histogram Comparison

- Which measure is best?
  - Depends on the application...
  - Euclidean distance is often not robust enough.
  - Both Intersection and  $\chi^2$  give good performance for histograms.
    - Intersection is a bit more robust.
    - $\chi^2$  is a bit more discriminative.
  - KL/Jeffrey works sometimes very well, but is expensive.
  - EMD is most powerful, but also quite expensive
  - There exist many other measures not mentioned here
    - e.g. statistical tests: Kolmogorov-Smirnov  
Cramer/Von-Mises
    - ...

# Summary: Recognition Using Histograms

- Simple algorithm
  1. Build a set of histograms  $H = \{h_i\}$  for each known object
    - More exactly, for each *view* of each object
  2. Build a histogram  $h_t$  for the test image.
  3. Compare  $h_t$  to each  $h_i \in H$ 
    - Using a suitable comparison measure
  4. Select the object with the best matching score
    - Or reject the test image if no object is similar enough.

**“Nearest-Neighbor” strategy**

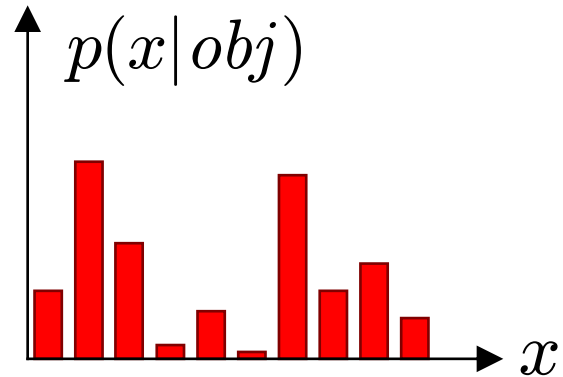
# Topics of This Lecture

- Object Recognition
  - Appearance-based recognition
  - Global representations
  - Color histograms
- Recognition using histograms
  - Histogram comparison measures
  - Histogram backprojection
  - Multidimensional histograms
  - Extension: colored derivatives



# Localization by Histogram Backprojection

- „Where in the image are the colors we‘re looking for?“
  - Idea: Normalized histogram represents probability distribution

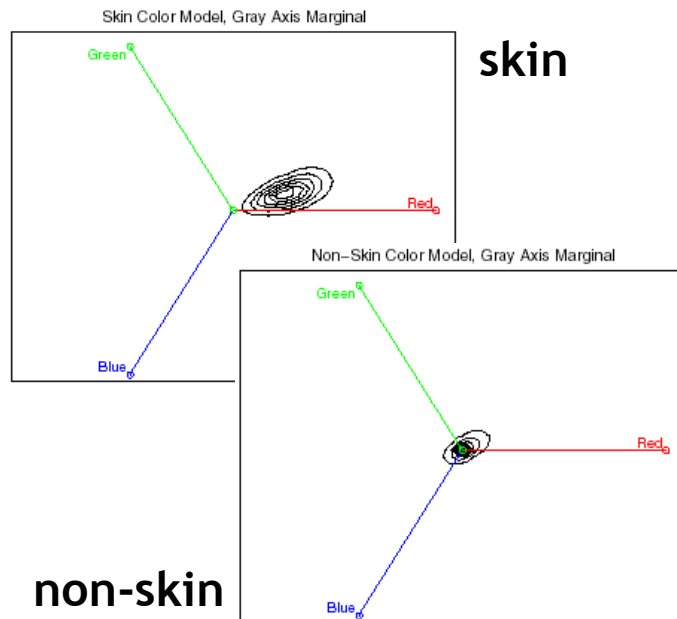


- Histogram backprojection
  - For each pixel  $x$ , compute the **likelihood** that this pixel color was caused by the object:  $p(x|obj)$ .
  - This value is projected back into the image (*i.e.* the image values are replaced by the corresponding histogram values).



# Color-Based Skin Detection

- Used 18,696 images to build a general color model.
- Histogram representation



M. Jones and J. Rehg, [Statistical Color Models with Application to Skin Detection](#), IJCV 2002.

# Discussion: Color Histograms

- Pros

- Invariant to object translation & rotation
- Slowly changing for out-of-plane rotation
- No perfect segmentation necessary
- Histograms change gradually when part of the object is occluded
- Possible to recognize deformable objects
  - E.g., a pullover

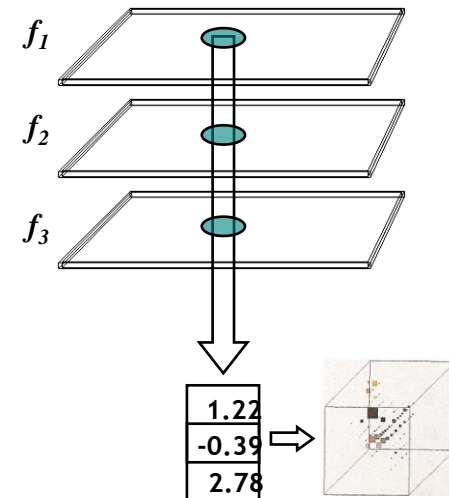
- Cons

- Pixel colors change with the illumination („color constancy problem“)
  - Intensity
  - Spectral composition (illumination color)
- Not all objects can be identified by their color distribution.



# Topics of This Lecture

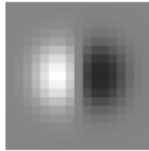
- Object Recognition
  - Appearance-based recognition
  - Global representations
  - Color histograms
- Recognition using histograms
  - Histogram comparison measures
  - Histogram backprojection
  - **Multidimensional histograms**
  - Extension: colored derivatives



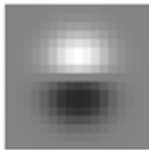
# Generalization of the Idea

- Histograms of derivatives

➤ Dx



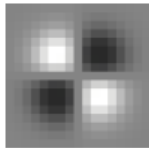
➤ Dy



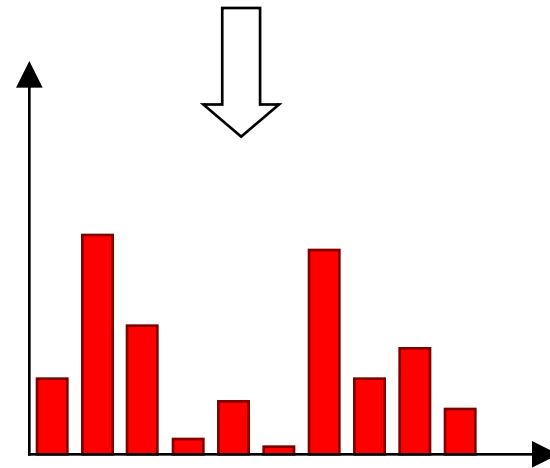
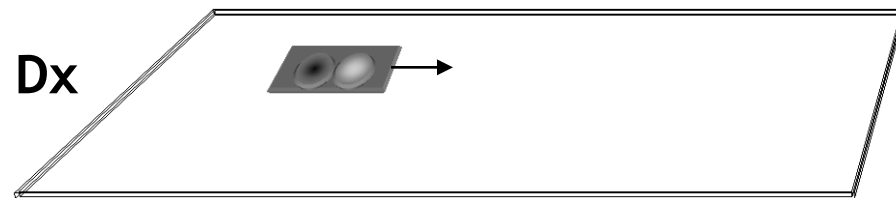
➤ Dxx



➤ Dxy



➤ Dyy



# General Filter Response Histograms

- Any local descriptor (e.g. filter, filter combination) can be used to build a histogram.

- **Examples:**

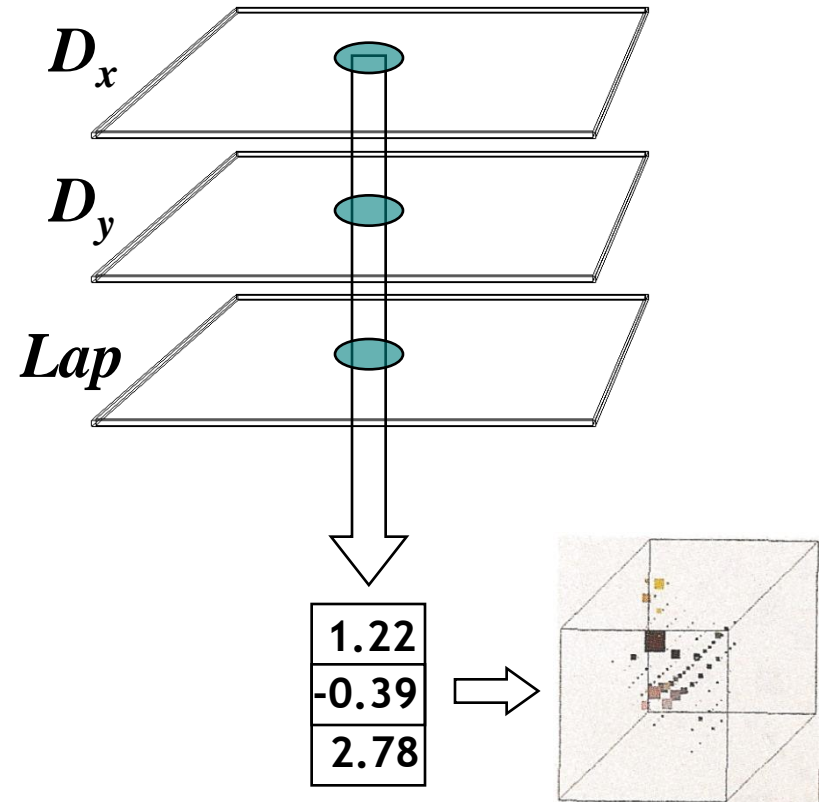
- Gradient magnitude  $Mag = \sqrt{D_x^2 + D_y^2}$

- Gradient direction  $Dir = \arctan \frac{D_y}{D_x}$

- Laplacian  $Lap = D_{xx} + D_{yy}$

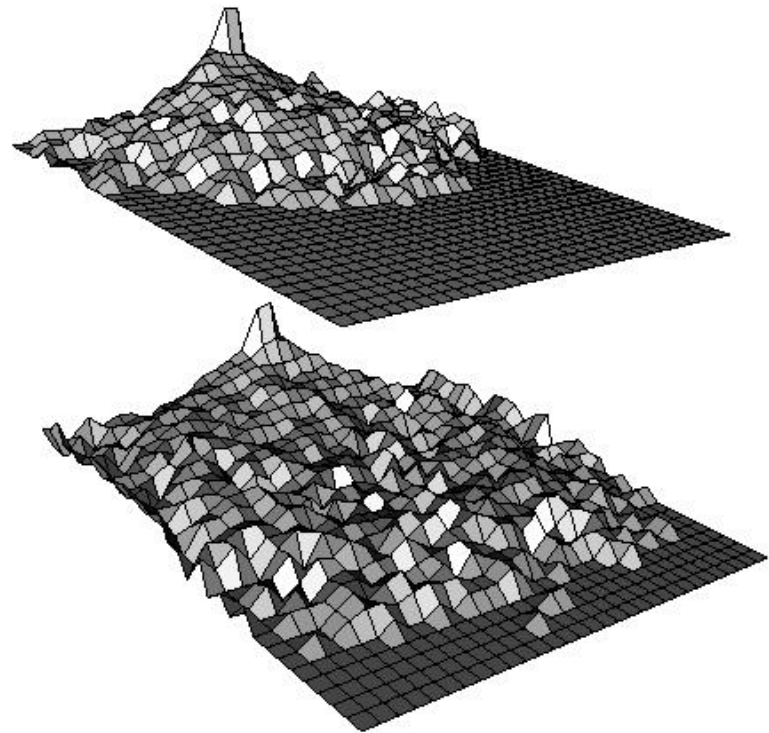
# Multidimensional Representations

- Combination of several descriptors
  - Each descriptor is applied to the whole image.
  - Corresponding pixel values are combined into one feature vector.
  - Feature vectors are collected in multidimensional histogram.



# Multidimensional Histograms

- Examples



# Multidimensional Representations

- Useful simple combinations

- $D_x$ - $D_y$

- Rotation-variant

- Descriptor changes when image is rotated.
      - Useful for recognizing oriented structures (e.g. vertical lines)

- *Mag-Lap*

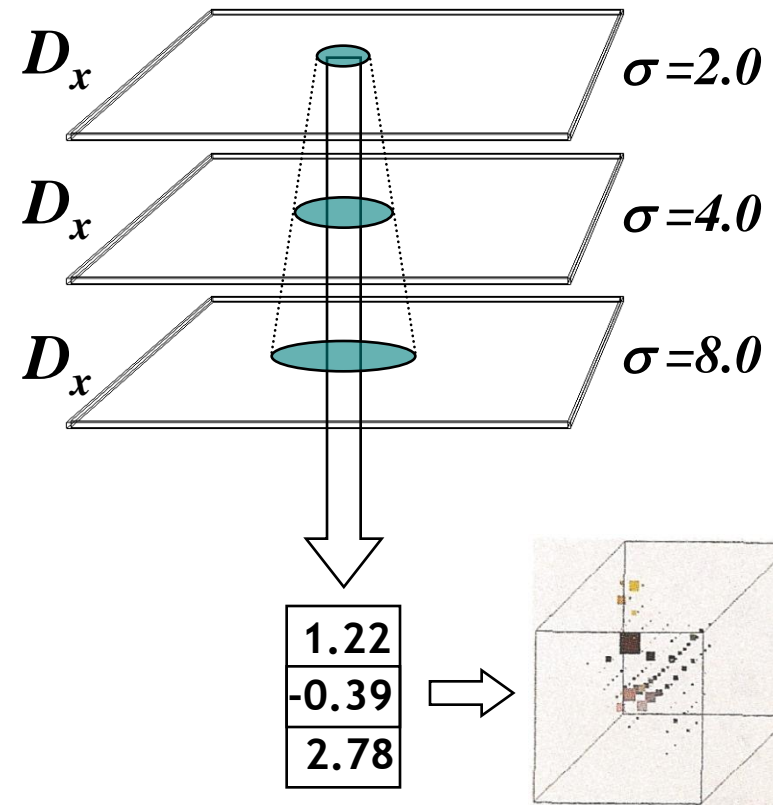
- Rotation-invariant

- Descriptor does *not* change when image is rotated.
      - Can be used to recognize rotated objects.
      - Less discriminant than rotation-variant descriptor.

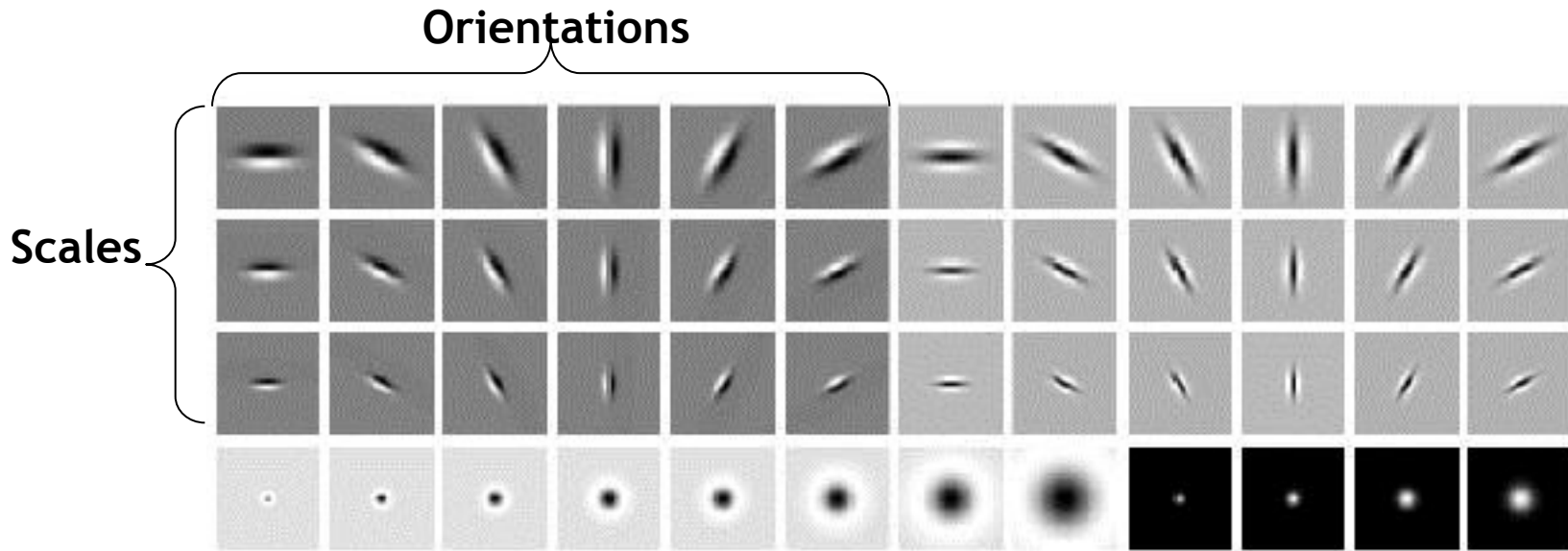


# Special Case: Multiscale Representations

- Combination of several scales
  - Descriptors are computed at different scales.
  - Each scale captures different information about the object.
  - Size of the support region grows with increasing  $\sigma$ .
  - Feature vectors capture both local details and larger-scale structures.



# Generalization: Filter Banks



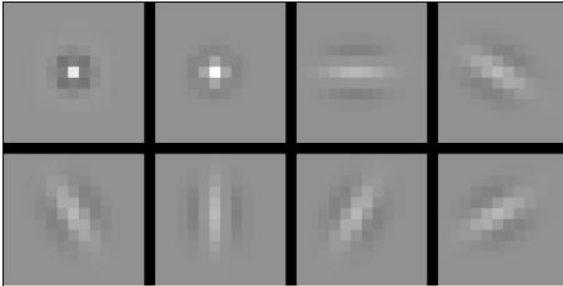
- What filters to put in the bank?
  - Typically we want a combination of scales and orientations, different types of patterns.

Matlab code available for these examples:

<http://www.robots.ox.ac.uk/~vgg/research/texclass/filters.html>



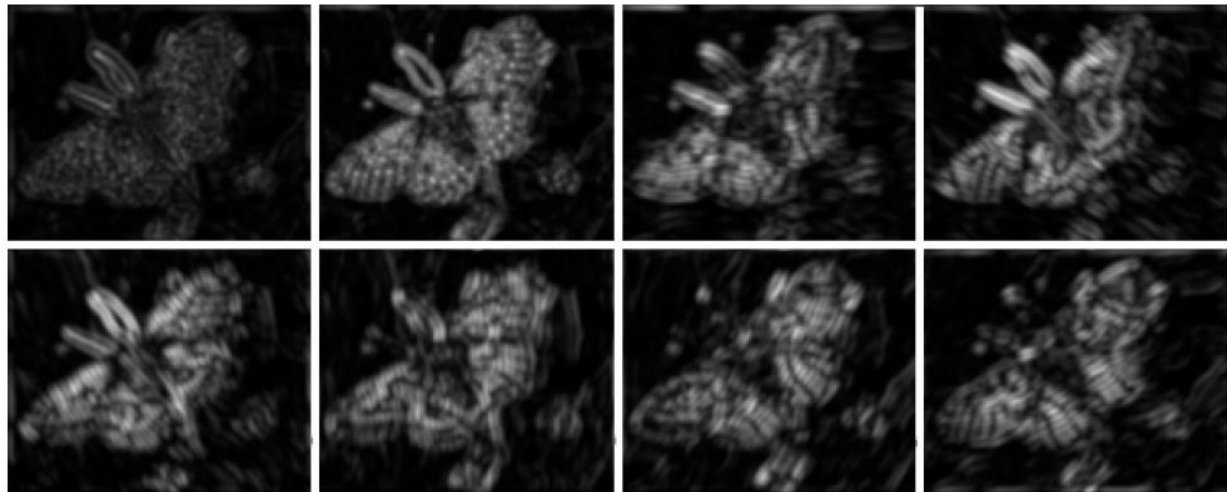
# Example Application of a Filter Bank



Filter bank of 8 filters



Input image

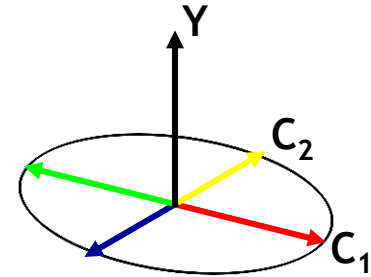


8 response images: magnitude  
of filtered outputs, per filter

# Extension: Colored Derivatives

- $YC_1C_2$  color space

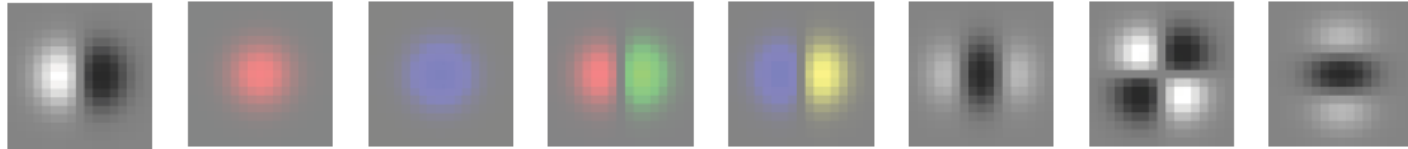
$$\begin{pmatrix} Y \\ C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} g_r & g_g & g_b \\ \frac{3g_g}{2} & -\frac{3g_r}{2} & 0 \\ \frac{g_b g_r}{g_r^2 + g_g^2} & \frac{g_b g_g}{g_r^2 + g_g^2} & -1 \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$



- Color-opponent space

- Inspired by models of the human visual system
- $Y \equiv$  intensity
- $C_1 \equiv$  red-green
- $C_2 \equiv$  blue-yellow

# Extension: Colored Derivatives



- **Generalization: derivatives along**
  - Y axis → intensity differences
  - $C_1$  axis → red-green differences
  - $C_2$  axis → blue-yellow differences
- **Feature vector is rotated such that  $D_y = 0$** 
  - Rotation-invariant descriptor

# Summary: Multidimensional Representations

- Pros

- Work very well for recognition.
- Usually, simple combinations are sufficient (e.g.  $D_x$ - $D_y$ , *Mag-Lap*)
- But multiple scales are very important!
- Generalization: filter banks

- Cons

- High-dimensional histograms      ⇒ lots of storage space
- Global representation              ⇒ not robust to occlusion

# Application: Brand Identification in Video



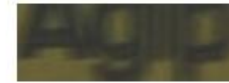
# Application: Brand Identification in Video



	→	0.76
	→	0.01
	→	0.51
	→	0.14
	→	0.29
	→	0.47



# Application: Brand Identification in Video



2%



3%



11%



0%



33%

false detection

# References and Further Reading

- Background information on histogram-based object recognition can be found in the following paper
  - B. Schiele, J. Crowley,  
*Recognition without Correspondence using Multidimensional Receptive Field Histograms.*  
International Journal of Computer Vision, Vol. 36(1), 2000.
- Matlab filterbank code available at
  - <http://www.robots.ox.ac.uk/~vgg/research/texclass/filters.html>